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On performance of $\overline{X}$ and R-charts under the assumption of moderateness rather than normality and with 3δ control limits rather than 3σ control limits

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Abstract

Mean deviation (δ) is a very good alternative of standard deviation (σ) as mean deviation is considered to be the most intuitively and rationally defined measure of dispersion. This fact can be very useful in the field of quality control to construct the control limits of the control charts. Most of the research work related to control charts for variables assume that the variable under study follows normal distribution and the control limits are laid at 3σ distance from the central line when σ is the standard error of the statistic being used for constructing the control chart. In this paper it has been assumed that the underlying distribution of the variable of interest follows moderate distribution proposed by Desai J.M. [1], which has mean deviation as scale parameter rather than the standard deviation. Further, the upper and lower control limits are set at 3δ distance from the central line where δ is the mean deviation of sampling distribution of the statistic being used for constructing the control chart. Also the performance analysis of the control charts is carried out with the help of OC curve analysis and ARL curve analysis.

Key words : Mean deviation, Standard deviation, Control charts, Normal distribution, Moderate distribution, Statistic, OC function, ARL curve.

1. Introduction:

A fundamental assumption in the development of control charts for variables is that the underlying distribution of the quality characteristic is normal. The normal distribution is one of the most important distributions in the statistical inference in which mean (μ) and standard deviation (σ) are the parameters of this distribution. Desai J.M. [1] has suggested an alternative of normal distribution, which is called moderate distribution. In moderate distribution mean (μ) and mean deviation (δ) are the pivotal parameter and it has properties similar to normal distribution. Under this statistical model with different values of μ and δ, one gets different moderate distributions. All moderate distributions are symmetric and bell-shaped. He has also prepared moderate table (similar to normal table) pertaining to the area under standard moderate curve. With the help of this table, it can be verified that the significant moderation takes place in the distribution of probability in normal distribution when for the fixed value of first degree dispersion (FDD); the dispersion parameter σ (the standard deviation) is replaced by another dispersion parameter, δ (mean deviation). For example, in standard normal distribution having FDD, K= σ =1, about 3 observations out of 10 are expected to lie outside the range μ ± K whereas in moderate distribution with K=δ =1, about 4 observations out of 10 are expected to lie outside the range μ ± K. Similarly, for normal distribution with FDD, K= σ =3, about 27 observations out of 10,000 are expected to lie outside the range μ ±3K whereas in moderate distribution with K=δ =3, about 167 observations out of 10,000 are expected to lie outside the range μ.± 3K. Therefore, in this paper under the assumption of moderateness, $\bar{X}$ and R charts are studied and the 3δ control limits are derived. The performance analysis of $\bar{X}$ and R charts under the assumption of moderateness against the under normality assumption has also been carried out through OC curve analysis and ARL curve analysis.

2. The concept of 3δ control limits in SQC charts:

Control limits are one of the key factors key factors in the quality control charts. Let T be any statistic representing some characteristic of quality which follows moderate distribution having parameters mean (μ) and mean error (mean deviation δ). In usual control charts based on normality assumption, UCL and LCL are set at the distance 3σ from central line mainly because in normal distribution σ is the scale parameter and it is a measure of dispersion. Since we are assuming moderateness instead of normality and the dispersion parameter of the moderate distribution is δ (which is equivalent to the dispersion parameter σ of normal distribution), under this assumption UCL and LCL should be set at 3δ distance from central line (i.e.3 times mean deviation of the sampling distribution of statistic T).

Let the mean of T be μ and M.E. of T (mean deviation of sampling distribution of T) is δ_T. Then, in the moderate distribution,

$$P(\mu-3\delta<X<\mu+3\delta) = 0.98332$$

i.e. in moderate distribution about 98.33 % of the observations lie within the interval $\mu \pm 3\delta_T$. Which means the probability that any observation will fall outside this range is 0.0167. i.e. less than 2% and so it is negligible.

Thus in the proposed control charts, under the moderateness assumption, three control limits for any statistic T should be determined as follows.

Central line (CL) = Expected value of T = μ

Lower Control Limit (LCL) = Mean of T - $3\delta_T = \mu - 3\delta_T$

Upper Control Limit (UCL) = Mean of T + $3\delta_T = \mu + 3\delta_T$

Such control limits may be called 3δ limits of the charts. It is proposed that if the expected standard is not predetermined, the average value of T from samples may be taken as central line. The value of mean error δ_T is also found out from different values of T. It is found that since δ provides exact average distance from mean and σ provides only an approximate average distance, 3δ limits are more rational as compared to 3σ limits.

3. 3δ control limits for R-chart when process mean deviation δ' is unknown:

Suppose a measurable quality characteristic of the product is denoted by X. Suppose that m samples, each of size n, are drawn at more or less regular interval of time from the production processes. These samples are known as subgroups, and for each of these subgroups the values of mean $\bar{X}$ and range R are obtained. Let the distribution of the variable X be moderate with mean μ and mean deviation δ, then, as proved by Desai J.M. [1], the distribution of $\bar{X}$ is also moderate with mean μ and mean deviation $\frac{\delta}{\sqrt{n}}$. Further, if the distribution of X is not moderate, and the number of units in each subgroup is 4 or more, then on the basis of central limit theorem for moderate distribution, it can be said that $\bar{X}$ follows moderate distribution.
The random variable W, which is called relative range, has following properties when it follows moderate distribution.

Relative range W $= \sqrt{\frac{2}{\pi}}\frac{R}{\delta'}$ (1)

Where δ' = predetermined process mean deviation.
Further,

E(W) = E($\sqrt{\frac{2}{\pi}}\frac{R}{\delta'}$) = d_2 (2)

$\therefore$E(R) = $\sqrt{\frac{\pi}{2}}\, d_2\delta'$ (3)

Hence, when δ' is not perdetermined

$$\delta' = \sqrt{\frac{2}{\pi}}\frac{E(R)}{d_2} = \sqrt{\frac{2}{\pi}}\frac{\bar{R}}{d_2} \quad (4)$$

Similarly, if δ_w and δ_R are mean deviation of W and R respectively then it can be seen that

$$\delta_w = \frac{\delta_R}{\delta'} = d_3 \quad \text{and} \quad \delta_R = \sqrt{\frac{2}{\pi}}\frac{d_3\bar{R}}{d_2} \quad (5)$$

Hence, the 3δ- control limits of R chart can be determined as follows.

Central line (C.L) = E(R)

= $\bar{R}$ (6)

Lower control limit (L.C.L) = E(R) - 3δ_R

= $\bar{R} - 3\sqrt{\frac{2}{\pi}}\frac{d_3\bar{R}}{d_2}$

= $(1 - 3\sqrt{\frac{2}{\pi}}\frac{d_3}{d_2})\bar{R}$

= $D_3'\bar{R}$ (7)

Where $D_3' = 1 - 3\sqrt{\frac{2}{\pi}}\frac{d_3}{d_2}$

Upper control limit (U.C.L) = E(R) + 3δ_R

= $\bar{R} + 3\sqrt{\frac{2}{\pi}}\frac{d_3\bar{R}}{d_2}$

= $(1 + 3\sqrt{\frac{2}{\pi}}\frac{d_3}{d_2})\bar{R}$

= $D_4'\bar{R}$, (8)

Where $D_4{}' = 1+3\sqrt{\frac{2}{\pi}}\frac{d_3}{d_2}$

The constants $D_3{}'$ and $D_4{}'$ are computed for n = 2 to 25 and are presented in table 1.

4. 3δ control limits for $\bar{X}$ -chart when process mean deviation δ' is unknown:

Central line (C.L) $= E(\bar{X})$

$$= \bar{\bar{X}} \quad (9)$$

Lower control limit(L.C.L) $= E(\bar{X}) - 3\delta_{\bar{X}}$

$$= \bar{\bar{X}} - 3\frac{\delta'}{\sqrt{n}}$$

$$= \bar{\bar{X}} - 3\frac{1}{\sqrt{n}}\sqrt{\frac{2}{\pi}}\frac{\bar{R}}{d_2}$$

$$= \bar{\bar{X}} - A_2{}'\bar{R}, \quad (10)$$

Upper control limit(U.C.L) $= E(\bar{X}) + 3\delta_{\bar{X}}$

$$= \bar{\bar{X}} + 3\frac{\delta'}{\sqrt{n}}$$

$$= \bar{\bar{X}} + 3\frac{1}{\sqrt{n}}\sqrt{\frac{2}{\pi}}\frac{\bar{R}}{d_2}$$

$$= \bar{\bar{X}} + A_2{}'\bar{R}, \quad (11)$$

Where $A_2{}' = \frac{3}{d_2\sqrt{n}}\sqrt{\frac{2}{\pi}}$ and its values are given in the table 1.

Table 1

Values of $D_3{}'$, D_3, $D_4{}'$, D_4, $A_2{}'$ and A_2 for different values of n						
n	$D_3{}'$	D_3	$D_4{}'$	D_4	$A_2{}'$	A_2
2	0	0	2.8097	3.267	1.4997	1.880
3	0	0	2.2553	2.575	0.8160	1.023
4	0	0	2.0228	2.282	0.5815	0.729
5	0.1111	0	1.8889	2.115	0.4603	0.577
6	0.1992	0	1.8008	2.004	0.3853	0.483
7	0.2628	0.076	1.7372	1.924	0.3343	0.419
8	0.3108	0.136	1.6892	1.864	0.2975	0.373
9	0.3489	0.184	1.6511	1.816	0.2688	0.337
10	0.3803	0.223	1.6197	1.777	0.2457	0.308
11	0.4064	0.256	1.5936	1.744	0.2151	0.285
12	0.4286	0.284	1.5714	1.716	0.2122	0.266
13	0.4476	0.308	1.5524	1.622	0.1986	0.249
14	0.4641	0.329	1.5359	1.671	0.1874	0.235
15	0.4790	0.348	1.5210	1.652	0.1779	0.223
16	0.4919	0.364	1.5081	1.636	0.1691	0.212

17	0.5038	0.379	1.4962	1.621	0.1619	0.203
18	0.5142	0.392	1.4859	1.608	0.1547	0.194
19	0.5238	0.404	1.4763	1.596	0.1492	0.184
20	0.5330	0.414	1.4670	1.586	0.1436	0.180
21	0.5414	0.425	1.4586	1.575	0.1380	0.173
22	0.5489	0.434	1.4511	1.566	0.1332	0.167
23	0.5559	0.443	1.4442	1.557	0.1292	0.162
24	0.5626	0.451	1.4374	1.548	0.1253	0.157
25	0.5690	0.459	1.4310	1.541	0.1220	0.153

5. Comparison of performance of $\bar{X}$ and R charts under the assumption of normality with 3σ- limits against that under the assumption of moderateness and 3δ-limits:

There are two commonly used methods for measuring and comparing the performance of control charts. One of the methods is to determine the Operating Characteristic (OC) curve of the charts. Another one is to determine the average run length (ARL).

It is very helpful to use the operating characteristic (OC) curve of a control chart to display its probability of type-II error. This would be an indication of the ability of the control chart to detect process shifts of different magnitudes. The OC Curve shows the probability that an observation will fall within the control limits given the state of the process. This is very much like finding power curves in hypothesis testing. Another measure of performance that is closely related to OC curve values is the run length. The run length is a random variable and is defined as the number of points plotted on the chart until an out-of-control condition is signaled.

5.1 Comparison of performance of R chart under the assumption of normality with 3σ- limits against that under the assumption of moderateness with 3δ-limits through OC curve analysis:

The two types of errors for control charts are defined as follows.

Let α = probability of type-I error of control charts.

= probability that the process is considered to be of out control when it is really in control,

Similarly,

β = probability of type-II error of control charts.

= probability that the process is considered to be in control when it is really out of control, Clearly, 1- β = probability of not committing type-II error of control charts. Thus lower value of β and higher value of (1- β) means more effectiveness (better performance) of control charts.

To determine the OC function for the R-chart, the distribution of the relative range W = $\sqrt{\frac{2}{\pi}}\frac{R}{\delta}$ is used. Suppose that the 'in-control' value of mean deviation is δ_0. If the mean deviation shifts from the 'in-control' value- say δ_0 to another value, say $\delta_1 > \delta_0$, then the probability of not detecting a shift to a new value of δ, say $\delta_1 > \delta_0$, on the first sample following the shift is,

$$\beta = P\{LCL \leq R \leq UCL/\delta_1 > \delta_0\} \quad (12)$$

Under the assumption of moderateness, since mean of R is $\bar{R}$ and mean error is $d_3\delta_1$ and since the upper and lower 3δ control limits are UCL= $D_4{}'\bar{R}$ and LCL =$D_3{}'\bar{R}$, where $D_3{}'$ and $D_4{}'$ are as determined in section 3, if for the R-chart β_{mR} is β under moderateness assumption then, the equation (12) can be written as follows,

$$\beta_{mR} = \emptyset'\left[\frac{UCL-\bar{R}}{d_3\delta_1}\right] - \emptyset'\left[\frac{LCL-\bar{R}}{d_3\delta_1}\right]$$

$$= \emptyset'\left[\frac{D_4{}'\bar{R}-\bar{R}}{d_3\delta_1}\right] - \emptyset'\left[\frac{D_3{}'\bar{R}-\bar{R}}{d_3\delta_1}\right]$$

$$= \emptyset'\left[\frac{\bar{R}(D_4{}'-1)}{d_3\delta_1}\right] - \emptyset'\left[\frac{\bar{R}(D_3{}'-1)}{d_3\delta_1}\right]$$

$$= \emptyset'\left[\frac{d_2\delta_0(D_4{}'-1)}{d_3\delta_1}\right] - \emptyset'\left[\frac{d_2\delta_0(D_3{}'-1)}{d_3\delta_1}\right]$$

$$= \emptyset'\left[\frac{d_2}{d_3}.\frac{(D_4{}'-1)}{\lambda_1{}'}\right] - \emptyset'\left[\frac{d_2}{d_3}.\frac{(D_3{}'-1)}{\lambda_1{}'}\right] \quad (13)$$

Where $\lambda_1{}' = \frac{\delta_1}{\delta_0}$ or $\delta_1 = \lambda_1{}'\delta_0$ and $\emptyset'$ denotes the standard moderate cumulative probability distribution. Equation (13) is called OC function of R chart. Similarly, if in the R-chart β_{nR} is β under normality assumption then, OC function of R chart under normality assumption can be determined as follows,

$$\beta_{nR} = \emptyset\left[\frac{d_2}{d_3}.\frac{(D_4-1)}{\lambda_1}\right] - \emptyset\left[\frac{d_2}{d_3}.\frac{(D_3-1)}{\lambda_1}\right] \quad (14)$$

Where $\lambda_1 = \frac{\sigma_1}{\sigma_0}$ or $\sigma_1 = \lambda_1\sigma_0$ and D_3 and D_4 are the constants being used in usual R-chart. Usually shift in the value of δ_0(or σ_0) is measured in terms of percentage of its 'in control' value. Thus δ_1= 1.1δ_0 means 10% shift in the value of δ_0, δ_1= 1.5δ_0 means 50% shift in the value of δ_1, δ = 2δ_0 means 100% shift in the value of δ_0. Hence $\lambda_1{}'$(and λ_1) are usually chosen in the range [1, 2].

To construct the OC curve for R chart under moderateness (or normality) assumption, β-value is plotted against $\lambda_1{}'$ (or λ_1) with various sample sizes n. These probabilities may be evaluated directly from equation (13) and (14).

For different sample sizes n and with three-delta limits (or three-sigma limits), for various values of $\lambda_1{}'$ (or λ_1) , β-values are calculated and OC curves are plotted as shown in figure-1.

Table 2

λ_1(or $\lambda_1{}'$)	n = 3		n = 4		n = 5		n = 8	
	β_{mR}	β_{nR}	β_{mR}	β_{nR}	β_{mR}	β_{nR}	β_{mR}	β_{nR}
1.0	0.9079	0.9706	0.9408	0.9891	0.9434	0.9951	0.9434	0.9974
1.2	0.8415	0.9379	0.8839	0.9682	0.8876	0.9813	0.8876	0.9876
1.4	0.7749	0.8969	0.8224	0.9363	0.8276	0.9564	0.8276	0.9676
1.6	0.7131	0.8529	0.7623	0.8978	0.7686	0.9234	0.7686	0.9400
1.8	0.6569	0.8079	0.7059	0.8557	0.7114	0.8857	0.7114	0.9050
2.0	0.6066	0.7621	0.6555	0.8122	0.6616	0.8447	0.6616	0.8664

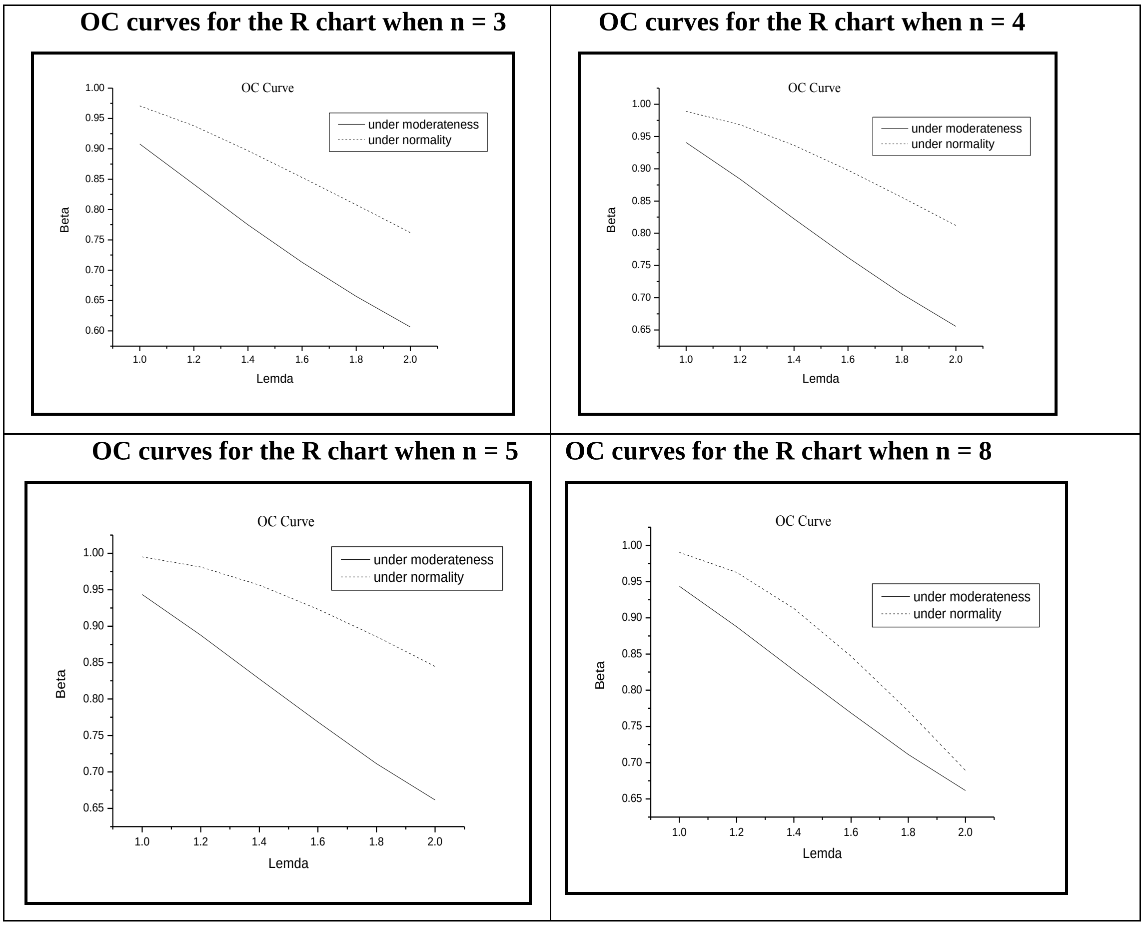


Figure 1

Form the above table 1 and figure 1, it is noticeable that for all values of n when $\lambda_1{}' = 1$(or $\lambda_0{}' = 1$) i.e even when there is no shift in the value of δ_0(or σ_0), β under moderateness assumption are always smaller than β under normality assumption, which indicates that R-chart under moderateness assumption and 3δ-limits is more effective than that of under normality assumption and 3σ-limits.

5.2 Comparison of performance of R chart under the assumption of normality with 3σ- limits against that under the assumption of moderateness with 3δ-limits through ARL curve analysis:

For a control chart the average run length (ARL) is the average number of points required to be plotted before a point indicates an out of control condition, that means when ARL is small, the chart is considered to be more effective. If the process observations are uncorrelated, then for any control chart, the ARL can be calculated easily from,

$$\text{ARL} = \frac{1}{p} = \frac{1}{1-\beta} \qquad (15)$$

Where p is said the probability that any point exceeds the control limits. This equation can be used to evaluate the performance of the control chart.

To construct the ARL curve for the R chart, ARL is plotted against the magnitude of the shift with various sample sizes n. To measure the effectiveness of the control charts under both the assumption moderateness and normality, the probabilities(β) may be evaluated directly from equations (13) and (14) and values of ARL are calculated from equation (15) and ARL curves are plotted.

For different sample sizes n and with three-delta limits (or three-sigma limits), for various values of $\lambda_1{}'$(or λ_1), ARLs are calculated and ARL curves are plotted as shown below.

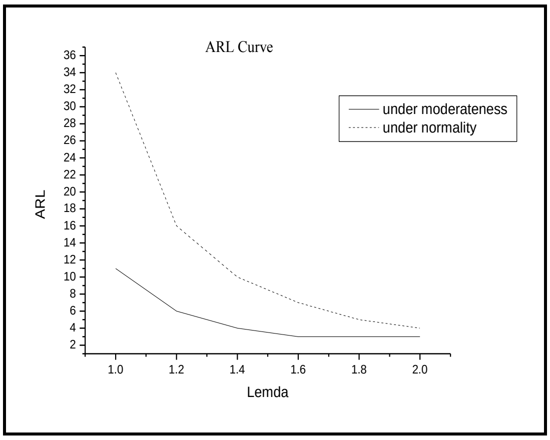
Table 3

λ_1(or $\lambda_1{}'$)	n = 3		n = 4		n = 5		n = 8	
	ARL_{mR}	ARL_{nR}	ARL_{mR}	ARL_{nR}	ARL_{mR}	ARL_{nR}	ARL_{mR}	ARL_{nR}
1.0	11	34	17	92	18	204	60	455
1.2	6	16	9	31	9	53	22	93
1.4	4	10	6	16	6	23	11	35
1.6	3	7	4	10	4	13	7	18
1.8	3	5	3	7	3	9	5	11
2.0	3	4	3	5	3	6	4	8

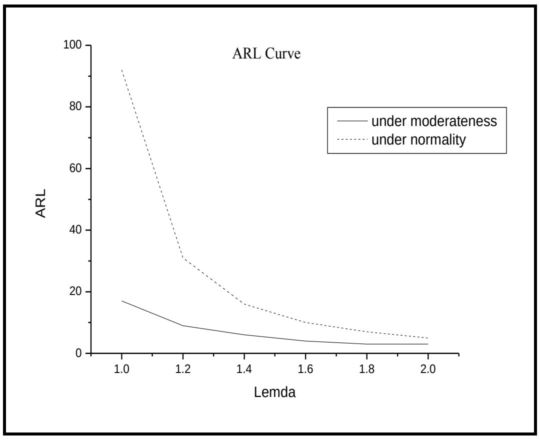
Where ARL_{mR} = ARL values under moderateness assumption for R-chart.

ARL_{nR} = ARL values under normality assumption for R-chart.

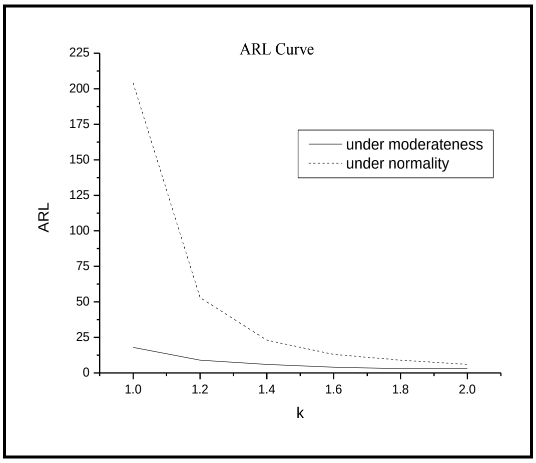
ARL curves for the R chart when n = 3



ARL curves for the R chart when n = 4



ARL curves for the R chart when n = 5



ARL curves for the R chart when n = 8

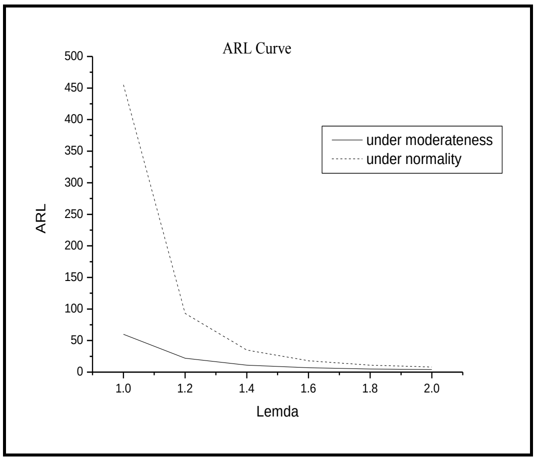


Figure 2

From the figures 2, it is clear that for all values of n, even when there is up to 100% shift in the value of δ_0(or σ_0), ARL under moderateness assumption are always smaller than ARL under normality assumption, which indicates that R chart under moderateness assumption is more effective than that of under normality assumption.

5.3 Comparison of performance of $\bar{\mathbf{X}}$ chart under the assumption of normality with 3$\boldsymbol{\sigma}$- limits against that under the assumption of moderateness with 3$\boldsymbol{\delta}$- limits through OC curve analysis:

Consider the OC curve for $\bar{X}$ chart with the mean deviation δ known and constant. If the mean shifts from the in-control value say, μ_0 - to another value $\mu_1 = \mu_0 + \text{k}\delta$, the probability of not detecting this shift on the first subsequent sample or the β risk, is,

$$\beta = \text{P}\left\{\text{LCL} \leq \bar{X} \leq \text{UCL} / \mu_0 = \mu_1 = \mu_0 + \text{k}\delta\right\} \qquad (16)$$

Here (kδ) 100 is the amount of shift in terms of percentage of FDD (average distance of values fromμ_0).Thus for k = 0.1, $\mu_1 = \mu_0 + 0.1\delta$, which means μ_0 shifts to μ_1 by 10% of values of FDD δ (average distance of values from μ_0). Normally, in the computation of β, the value of k is chosen in the range [0, 1]. Obviously $\mu_1 = \mu_0$ means $\delta = 0$ and therefore no shift in the value of process mean μ_0 and $\mu_1 = \mu_0 + \delta$ means the shift in the value of μ_0 is equal to 100% of the value of δ.

For $\overline{X} \sim M\,(\mu, \frac{\delta}{\sqrt{n}})$, the upper and lower control limits are UCL= μ_0 + L $\frac{\delta}{\sqrt{n}}$ and LCL = μ_0-L $\frac{\delta}{\sqrt{n}}$, if for the $\overline{X}$-chart β_{mX} is β under moderateness assumption then, equation (16) can be written as follows,

$$\beta_{mX} = \emptyset'\left[\frac{UCL-(\mu_0+k\delta)}{\frac{\delta}{\sqrt{n}}}\right] - \emptyset'\left[\frac{LCL-(\mu_0+k\delta)}{\frac{\delta}{\sqrt{n}}}\right]$$

$$= \emptyset'\left[\frac{\mu_0 + L\frac{\delta}{\sqrt{n}}-(\mu_0+k\delta)}{\frac{\delta}{\sqrt{n}}}\right] - \emptyset'\left[\frac{\mu_0 - L\frac{\delta}{\sqrt{n}}-(\mu_0+k\delta)}{\frac{\delta}{\sqrt{n}}}\right]$$

Where $\emptyset'$ denotes the standard moderate cumulative distribution. This reduces to,

$$\beta_{mX} = \emptyset'(L- k\sqrt{n}) - \emptyset'(-L-k\sqrt{n}) \tag{17}$$

Similarly, for $\overline{X} \sim N\,(\mu, \sigma = \sqrt{\frac{\pi}{2}}\frac{\delta}{\sqrt{n}})$ i.e. under normality assumption if β_{nR} is β under normality assumption then, (17) can be written as,

$$\beta_{nX} = \emptyset(L-k\sqrt{n}) - \emptyset(-L- k\sqrt{n}), \tag{18}$$

Where $\emptyset$ denotes the standard normal cumulative distribution.

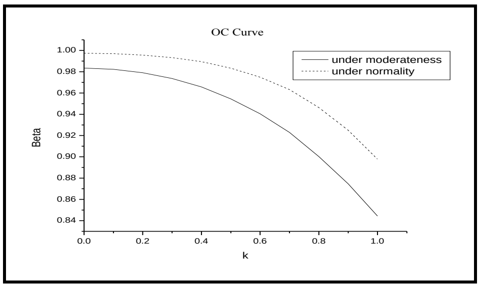
To construct the OC curve for the $\overline{X}$ chart, β-risk is plotted against the magnitude of the shift with various sample sizes n. To measure the effectiveness of the control charts under both the assumption moderateness and normality, the probabilities may be evaluated directly from equation (17) and (18) and OC curves are plotted.

For different sample sizes n and L = 3, for various values of k, β-risks are calculated under both the assumption and OC curves are plotted as shown below, where
β = P (not detecting a shift of kδ in the mean on the first sample following the shift.)

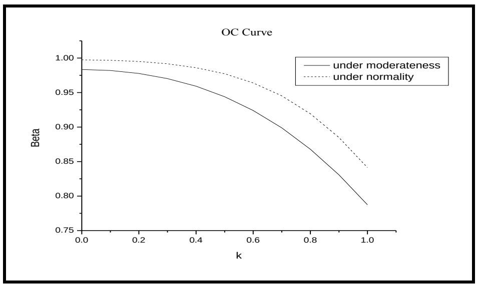
Table 4

k	n = 3		n = 4		n = 5		n = 8	
	β_{mX}	β_{nX}	β_{mX}	β_{nX}	β_{mX}	β_{nX}	β_{mX}	β_{nX}
0.0	0.9834	0.9974	0.9834	0.9974	0.9834	0.9974	0.9834	0.9974
0.1	0.9823	0.9970	0.9820	0.9968	0.9816	0.9967	0.9806	0.9962
0.2	0.9790	0.9956	0.9777	0.9950	0.9761	0.9944	0.9715	0.9924
0.3	0.9736	0.9932	0.9702	0.9917	0.9668	0.9900	0.9558	0.9842
0.4	0.9657	0.9895	0.9592	0.9860	0.9529	0.9825	0.9317	0.9692
0.5	0.9544	0.9833	0.9440	0.9772	0.9327	0.9699	0.8975	0.9440
0.6	0.9405	0.9749	0.9241	0.9640	0.9070	0.9515	0.8501	0.9031
0.7	0.9230	0.9632	0.8989	0.9452	0.8730	0.9236	0.7920	0.8461
0.8	0.9003	0.9463	0.8679	0.9192	0.8327	0.8868	0.7225	0.7703
0.9	0.8746	0.9250	0.8307	0.8849	0.7851	0.8389	0.6401	0.6736
1.0	0.8444	0.8979	0.7874	0.8412	0.7278	0.7763	0.5538	0.5674

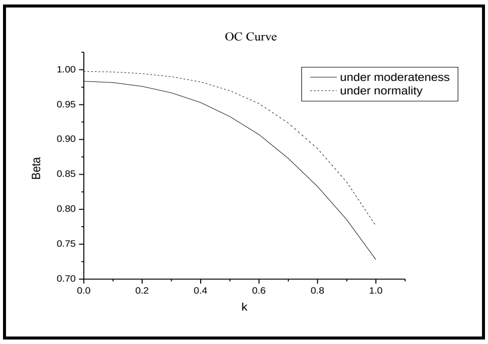
OC curves for the $\overline{X}$ chart when n = 3



OC curves for the $\overline{X}$ chart when n = 4



OC curves for the $\overline{X}$ chart when n = 5



OC curves for the $\overline{X}$ chart when n = 8

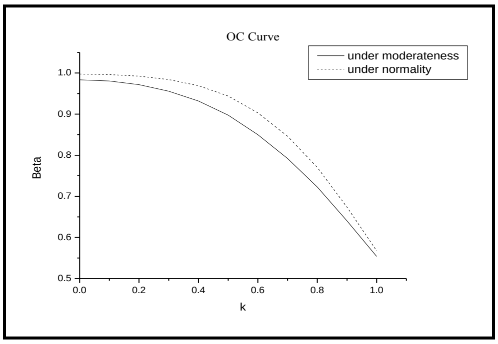


Figure 3

As noted earlier, from the table 3 and figure 3, it is noticeable that for k=0, i.e. when there is no shift in the process mean for any sample size, the value of β under moderateness assumption is always smaller than in the case of normality assumption, which means under moderateness assumption $\overline{X}$ chart is always more effective than in case of normality assumption. Figure (4) also indicates that for the most commonly used sample size (i.e up to 10), the chart under moderateness assumption are more effective towards detecting the shift in the value of μ_0 than the charts under normality assumption because value of β is lower or (1- β) is higher in former case as compared to the later case.

5.4 Comparison of performance of $\overline{X}$ chart under the assumption of normality with 3σ- limits against that under the assumption of moderateness with 3δ-limits through ARL curve analysis:

To construct the ARL curves for the $\overline{X}$ chart, ARL is plotted against the magnitude of the shift with various sample sizes n. To measure the effectiveness of the control charts under both the assumption moderateness and normality, the probabilities(β) may be evaluated directly from equation (17) and (18) and values of ARL are calculated from equation (15) and ARL curves are plotted.

For different sample sizes n and with three-delta limits (L = 3) and three-sigma limits (L=3), for various values of k, ARLs are calculated and ARL curves are plotted as shown below.

Table 5

k	n = 3		n = 4		n = 5		n = 8	
	ARL_{mX}	ARL_{nX}	ARL_{mX}	ARL_{nX}	ARL_{mX}	ARL_{nX}	ARL_{mX}	ARL_{nX}
0.0	60	382	60	385	60	385	60	385
0.1	56	333	56	313	54	303	52	263
0.2	48	227	45	200	42	179	35	132
0.3	38	147	34	120	30	100	23	63
0.4	29	95	25	71	21	57	15	32
0.5	22	60	18	44	15	33	10	18
0.6	17	40	13	28	11	21	7	10
0.7	13	27	10	18	8	13	5	6.
0.8	10	19	8	12	6	9	4	4
0.9	8	13	6	9	5	6	3	3
1.0	6	10	5	6	4	4	2	2

Where ARL_{mX} = ARL values under moderateness assumption for $\bar{X}$-chart.

ARL_{nX} = ARL values under normality assumption for $\bar{X}$-chart.

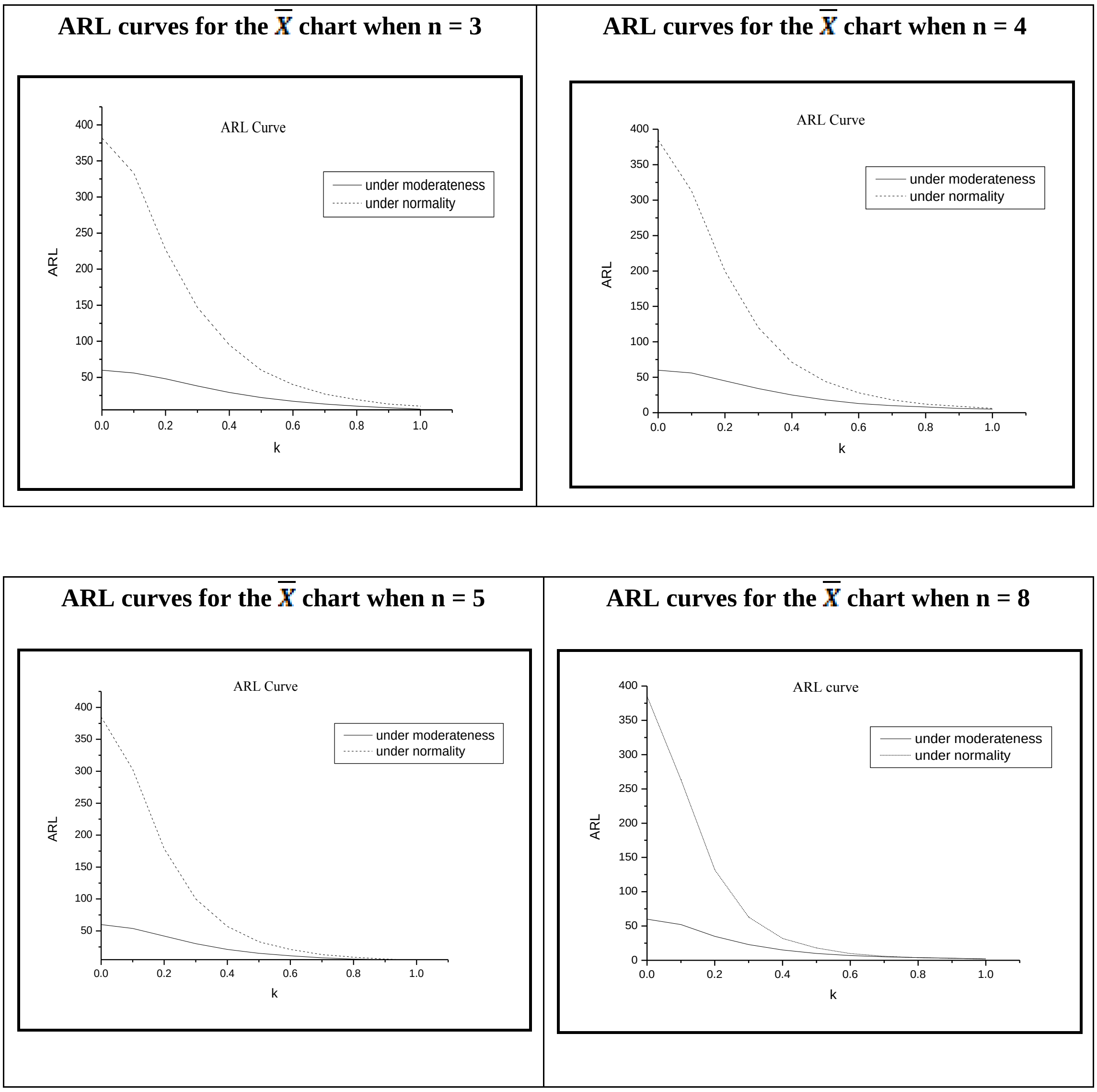


Figure 4

From the figures 4, it is clear that for all values of n, even when there is up to 100% shift in the value of δ_0(or σ_0), ARL under moderateness assumption are always smaller than ARL under normality assumption, which indicates that $\bar{X}$- chart under moderateness assumption is more effective than that of under normality assumption.

Summary

On the basis of OC curves analysis and ARL curves analysis, it is found that $\bar{X}$ and R charts under moderateness assumptions and having 3δ limits rather than 3σ limits are always more effective (perform better) than the charts under normality assumptions and having usual 3σ limits.

So it is recommended that the $\bar{X}$ and R control charts under moderateness assumption, and having 3δ limits rather than 3σ limits, should always be preferred over the $\bar{X}$ and R control charts under normality assumptions and having usual 3σ limits.

References

[1] Desai J.M. (2011): Alternatives of Normal Distribution and Its Related Distributions in which Mean and Mean Deviation Are the Pivotal Parameters and Their Application. Unpublished Ph.D. thesis submitted to Veer Narmad South Gujarat University, Surat