Vol.4,No.1, July, 2015
256 - 270,ISSN:0975-5446 [VNSGU JOURNAL OF SCIENCE AND TECHNOLOGY]

# Moderate Distribution: A modified normal distribution which has Mean as location parameter and Mean Deviation as scale parameter

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## Abstract:

In this paper the normal distribution is modified by replacing its scale parameter, the standard deviation, by another scale parameter which turns out to the mean deviation of the modified normal distribution. This modified normal distribution is named as 'Moderate Distribution'. Like Standard Normal Distribution, Standard Moderate Distribution is also defined. Properties of these distributions are reported and are compared with similar properties of normal distribution and the Standard Normal Distribution respectively. It has been shown that the distribution of probability in the proposed distribution is indeed moderate as compared to the normal distribution.

**Key words:** Normal Distribution, Standard Normal Distribution, Moderate Distribution, Standard Moderate Distribution, Standard Deviation, Mean Deviation.

## 1. Introduction:

The normal distribution is one of the most important distributions in the statistical inference and the standard deviation (S.D. or $\sigma$) is a pivotal parameter of this distribution. Therefore, S.D. as a measure of dispersion has assumed enormous importance even though the Mean Deviation (M.D. or $\delta$) is the most intuitively and rationally defined measure of dispersion and has several inherently superior merits over the S.D. Hence, it is strongly believed that if it is possible to find the probability distribution which is equally important as normal distribution is and which has dispersion parameter as M.D. then M.D. can also become as popular measure of dispersion as (or even more popular measure of dispersion than) S.D.

There is one such probability distribution known as Laplace distribution. But its properties are quite different from normal distribution. Moreover, the theories regarding inference related to its pivotal parameters mean and mean deviation are also not available. Therefore, the authors of this paper felt that if the debate about the relative merits of M.D. over S.D. restarted by

Gorard [1] is to be taken in the right direction and the use of mean deviation is to be popularized in common application then an important requirement is to find out a probability distribution which is a sound alternative of normal distribution, which has (i) mean and mean deviation as pivotal parameters and (ii) properties similar to normal distribution. Further, along with such an alternative of normal distribution if one can also find out alternatives of distributions related to normal distribution, such as chi-square distribution, t-distribution, F-distribution etc., then all the theories of statistical inference which are dependent on normal distribution and its related distribution can have equally sound alternatives. One of the authors of this paper has carried out this task in his Ph.D. thesis. In the present paper the moderate distribution and its important properties studied in his unpublished thesis have been presented.

## 2. Nature of moderation desired in an alternative of Normal distribution and a rationale for naming that alternative as 'Moderate Distribution':

Suppose a random variable X follows normal (or Gaussion) distribution. Then its p.d.f. is

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ,-\infty < x < \infty \quad ,-\infty < \mu < \infty, \quad \sigma > 0$$

where $\mu$ and $\sigma$ are respectively the location and scale parameters of the distribution. These two parameter $\mu$ and $\sigma$ of this distribution are its mean and standard deviation respectively.

Under this statistical model, with different values of $\mu$ and $\sigma$, one can get different normal distributions. All normal distributions are symmetric (i.e. skewness measure $\mu_3$ is zero), bell-shaped and coefficient of kurtosis $\beta_2$ is 3 irrespective of the value of $\mu$ and $\sigma$.

The normal distribution with parameters $\mu = 0$ and $\sigma = 1$, which is known as standard normal distribution, has its special importance among the normal distributions.

The graph of the normal distribution depends on two factors - the mean and the standard deviation. The mean of the distribution determines the location of the center of the graph, and the standard deviation determines the height and width of the graph. As shown below, when the standard deviation is large, the curve is relatively short and wide; when the standard deviation is small, the curve is relatively tall and narrow.

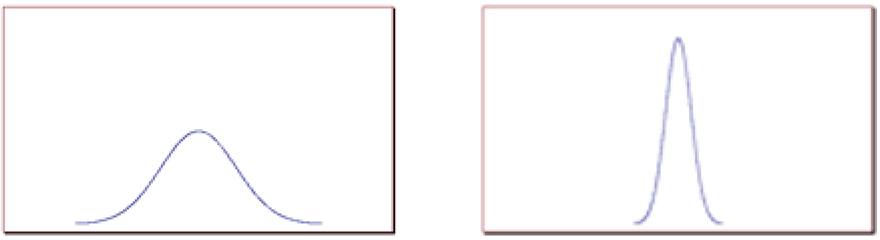


Figure 2.1 Curves of the normal distribution with different values of S.D.

The curve on the left is shorter and wider than the curve on the right, because the curve on the left has a bigger standard deviation.

About this most widely used distribution in practice, Hampal [2] has observed that small deviations from normality occur very frequently (almost always) in practice and Gorard [1] has argued that, in general, our observed distributions tend to be longer-tailed, having more extreme scores, then would be expected under ideal assumption of normality. The obvious interpretation of this argument of Gorard is that, in practice, in observed distributions, the dispersion is higher than expected and extreme scores have relatively higher chance of occurrence than expected under the assumption of normality. Thus, when normality is considered as an ideal assumption, there is a need for some moderation in allocation of chance of occurrence (probability) so that extreme scores get somewhat higher chance of occurrence than they get under the assumption of normality. In normal distribution if this is to be achieved then we should have relatively higher value of dispersion. In other words corresponding to any normal distribution we should be able to modify it to define another normal distribution which has same mean but relatively higher value of standard deviation so that the curve of modified distribution becomes little flatter as compared to that of original distribution indicating somewhat even distribution of probability.

Keeping in mind the discussion on issues related to the use of mean deviation, standard deviation and normal distribution in literature and particularly in the light of above comments of Hampal [2] and Gorard [1] related to observed distributions in practice having relatively longer tails as compared to the normal distribution, it is natural to think about whether or not we can have a longer tailed normal like distribution which has mean deviation as scale parameter and there is moderateness in allocation of probability of occurrence, in the sense that it has relatively lower chance of occurrence allocated to the values around mean and somewhat higher chance of occurrence allocated to its distant values located towards the tails. The answer to the later part of the question lies in moderating normal distribution by moderating, actually increasing, its dispersion.
Thus, the answer to the whole problem mentioned above seems to lie in moderating the normal distribution in such a way that in the moderated normal distribution the following two objectives are fulfilled.

(i) Mean deviation becomes the scale parameter in place of the standard deviation.
(ii) For given value of first degree dispersion (FDD), the values of $\delta$ and $\sigma$ become larger than their respective values in normal distribution.

In the following section 3.1 such a distribution with parameter $\mu$ and mean deviation $\delta$ is proposed which may be called moderated normal distribution or just **moderate distribution**. The authors of this paper have preferred to call it 'moderate distribution' because a meaning of "normal" (or "normality") is also contained in a meaning of "moderate" (or "moderateness") and also due to the reasons discussed in the section 3.2.

## 3. Proposed moderated (or alternative) normal distribution and further justification for naming it as 'Moderate Distribution

### 3.1 The proposed alternative (or moderated) normal distribution named as moderate distribution:

Suppose the p.d.f. of a distribution of a random variable X is defined as

$$f(x) = \frac{1}{\pi\delta} e^{-\frac{1}{\pi}\left(\frac{x-\mu}{\delta}\right)^2}, -\infty < x < \infty, \quad -\infty < \mu < \infty, \delta > 0 \quad . \tag{3.1.1}$$

Then, the random variable X may be said to be following **moderate distribution** with parameters $\mu$ and $\delta$ and may be denoted as
X ~ M ($\mu$, $\delta$).

It can very easily be proved that

(i) $$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{3.1.2}$$

(ii) Mean = E(X) = $\mu$ (3.1.3)

(iii) $$\text{Mean deviation} = E\left[|X-\mu|\right] = \delta \tag{3.1.4}$$

(iv) Standard deviation $= \sigma = \sqrt{\frac{\pi}{2}}\delta$ (3.1.5)

Note: It may be noted that the relationship between $\sigma$ and $\delta$ is same as that in the normal distribution.

(v) $\text{M.G.F.} = M_X(t) = e^{\mu t + \frac{\pi}{4}\delta^2 t^2}$ (3.1.6)

(vi) $f(\mu - x) = f(\mu + x)$ (3.1.7)

Thus, the distribution of a random variable X having p.d.f. as defined in (3.1.1) has location parameter as mean $\mu$ and scale parameter as mean deviation $\delta$ which means that the first objective of proposing this distribution is achieved.
To verify that the second objective of proposing the moderate distribution is also achieved, suppose the FDD is fixed at constant *'K'* units. Then for mean $\mu$ and FDD = *K*, the normal distribution is $N(\mu, \sigma = K)$ and the moderate distribution is $M(\mu, \delta = K)$.
Further, for normal distribution

$$\delta = \sqrt{\frac{2}{\pi}}\sigma = \sqrt{\frac{2}{\pi}}K$$

and for moderate distribution

$$\sigma = \sqrt{\frac{\pi}{2}}\delta = \sqrt{\frac{\pi}{2}}K.$$

Hence, corresponding to the values of $\sigma$ and $\delta$ in $N(\mu, \sigma = K)$, viz. $\sigma = K$ and $\delta = \sqrt{\frac{2}{\pi}}K$, we get in $M(\mu, \delta = K)$ the values of $\sigma$ and $\delta$ as

$$\sigma = \sqrt{\frac{\pi}{2}}K > K \text{ and } \delta = K > \sqrt{\frac{2}{\pi}}K.$$

Therefore, for given value of FDD, the values of $\delta$ and $\sigma$ in moderate distribution are larger than their respective values in normal distribution ( $\sigma$ has increased from *K* to $\sqrt{\frac{\pi}{2}}K$ and $\delta$ has increased from $\sqrt{\frac{2}{\pi}}K$ to *K*), which means that the second objective of proposing this distribution is also achieved.

**3.2 More justification for naming the modified distribution as Moderate Distribution:**

Besides the fact that a meaning of the word 'normal' is also contained in a meaning of the word 'moderate'; the fact that the modified normal distribution indeed has moderate (relatively more even) distribution of total probability (total area under the curve) over the values of the random variable may also be considered as justification for naming the modified normal distribution as moderate distribution. This is explained in points (a) and (b) given below.

(a) When the value of first degree dispersion is fixed at constant k, corresponding to the distribution $N(\mu, \sigma = K)$, having mean deviation $\delta = \sqrt{\frac{2}{\pi}}K$, the distribution

$M(\mu,\delta=K)$, having standard deviation $\sigma=\sqrt{\frac{\pi}{2}}K$, is nothing but a normal distribution in which the total probability ( area under the curve) is moderated (spread relatively evenly) by increasing the value of S.D. from K to $\sqrt{\frac{\pi}{2}}K$ ( i.e. by $(\sqrt{\frac{\pi}{2}}-1)\,100\% \cong 25\%)$, which also means increasing the value of M.D. from $\sqrt{\frac{2}{\pi}}K$ to K ( i.e. by $(1-\sqrt{\frac{2}{\pi}})100\% \cong 20\%)$. Here, the moderation of distribution of total probability over the values of the random variable means, for given K, the values of the variable having relatively higher(or lower) chance of occurrence in $N(\mu,\sigma=K)$ have relatively lower(or higher) chance of occurrence in $M(\mu,\delta=K)$. This fact can be seen to be very clearly reflected in the curves of the two distributions if they are plotted on the same graph. It can be seen in the graph given below that for mean zero and *K=1*, the curve of moderate distribution has moderate spread as compared to the spread of normal distribution curve.

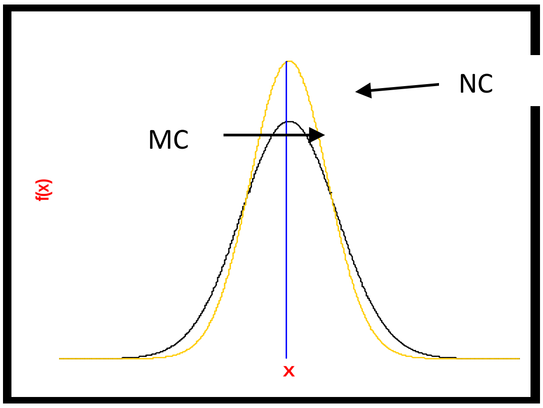



Figure.3.1 Normal curve (NC) and Moderate curve (MC)

(b) For empirical evidence in this regards one can consider the case of $N(0,\sigma=1)$ and $M(0,\delta=1)$ presented in the following table.

Table: 3.1 Comparison of area under Standard normal curve and Standard moderate curve

| ( a, b ) | Appro. P(a < X < b) in N(0,σ =1) | Appro. P(a < X < b) in M(0,δ =1) |
|---|---|---|
| (-1,1) | 0.68 | 0.58 |
| (-2,-1) or (1,2) | 0.14 | 0.16 |
| (-3,-2) or (2,3) | 0.02 | 0.05 |

It is clear from the table 3.1 that in the range (-1,1), i.e. around mean, a random variable following N(0,1) has relatively higher (about 10% higher) chance of occurrence than the one following M(0,1). However, outside this range, the chance of occurrence for the variable following M(0,1) has relatively higher chance of occurrence than the one following N(0,1). It is easily noticeable that the difference of about 10% in the range (-1,1) is spread out quite gradually outside this range indicating that the distribution of difference in the range is spread fairly evenly. This fact is elaborated even further in the section 6 of this paper.

**3.3. Justification for two separate distributions**:

It may be noted that, these two distributions (i.e. normal distribution and moderate distribution) are connected by the 'golden bridge' $\sigma=\sqrt{\frac{\pi}{2}}\delta$. In normal distribution, if we take

$(\frac{\pi}{2}\delta^2)$ in place of $\sigma^2$ then we get moderate distribution with parameters $\mu$ and $\delta$ and in moderate distribution if we take $(\frac{2}{\pi}\sigma^2)$ in place of $\delta^2$ then we get normal distribution with parameters $\mu$ and $\sigma$. Therefore, one can argue that when this moderate distribution (which only differs by the scale parameter) and its related properties can always be derived using normal distribution, why should one study this distribution separately?

The justification for treating these two distributions as different normal distributions is as follows:

(i) As noted earlier a normal distribution with S.D. = $K$ is different from the one with M.D. = $K$ because normal distributions with S.D. = $K$ and S.D. $= \sqrt{\frac{\pi}{2}}K$ are two different normal distributions.

(ii) The moderate distribution with mean deviation as a pivotal parameter will surely bring out the hither to suppressed importance of mean deviation as a measure of dispersion.

(iii) It will help in deriving the distributions related to (dependent on) proposed alternative of normal distribution (i.e. moderate distribution) which can be useful as alternatives of distributions like $\chi^2$-distribution, t-distribution, F-distribution etc.

(iv) It will also help in finding appropriate estimators for mean deviation and in developing related theory of statistical inferences and in general will promote the use of M.D. in practice.

## 4. Standard Moderate Distribution:

### 4.1 Definition and its important properties:

The standardized form of any distribution is the form that has location parameter zero and scale parameter one.

Suppose X ~ M ($\mu,\delta$).

Then, the variable Ƶ defined as

$$Ƶ = \frac{X-\mu}{\delta}$$

has the p.d.f. defined as

$$g(ƶ) = \frac{1}{\pi} e^{-\frac{1}{\pi}ƶ^2}, -\infty < ƶ < \infty \tag{4.1.1}$$

and this variable Ƶ may be called standard moderate variable. Also, its distribution may be called standard moderate distribution. In notations, one can write it as Ƶ ~ M (0,1).

It can be seen that

(i) Mean = E(Ƶ) = 0 (4.1.2)

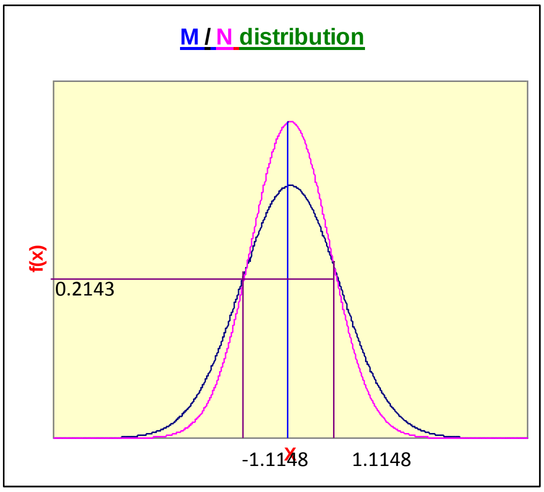
(ii) Mean deviation = $E[|Ƶ-\mu|] = 1$ (4.1.3)

(iii) Standard deviation = $\sqrt{\frac{\pi}{2}}$ (4.1.4)

(iv) $M_{Ƶ}(t) = e^{\frac{\pi}{4}t^2}$ (4.1.5)

### 4.2 Comparison between standard Normal Distribution and Standard Moderate Distribution:

Various properties of the two distributions as reflected in the probability curves of their standardized forms are easily noticeable in the following graphs.



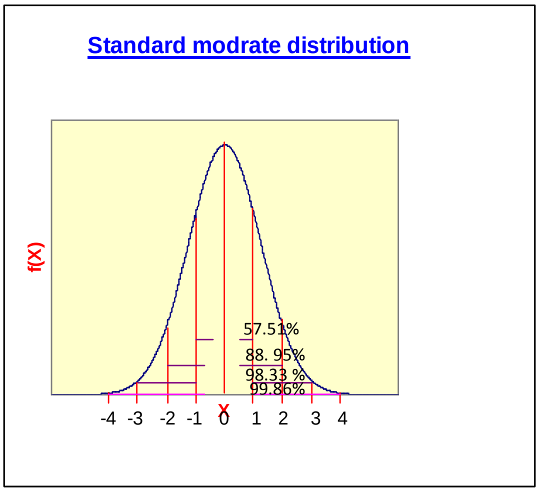




Figure. 4.1 Comparison of SNC and SMC Fig. 4.2 Area under SMC

For the comparison of SMD with SND the following points may be noted.

1. At the point X = 0, for standard normal distribution (SND) $f(x) = \frac{1}{\sqrt{2\pi}} = 0.39894228$ and for standard moderate distribution (SMD) $f(x) = \frac{1}{\pi} = 0.318309886$. Thus, the standard moderate curve (SMC) has lower peak than that of standard normal curve (SNC).

2. SNC and SMC are intersecting at the points (-1.114777015, 0.214316275) and (1.114777015, 0.214316275).

This means, the probability of occurrence for the values around -1.114777015 and 1.114777015 are approximately same for these two distributions.

3. If we consider the areas under these two curves and the line passing through the points (-1.114777015, 0.214316275) and (1.114777015, 0.214316275) then it is easily noticeable that the standard moderate curve has lower and broader peak as compared to higher and narrower peak of standard normal curve.

4. For standard normal distribution

$$P(-1.114777015 < X < 1.114777015) = 0.7350539$$

and for standard moderate distribution

$$P(-1.114777015 < X < 1.114777015) = 0.6262459.$$

Hence, about 74% of values of standard normal variable fall in the range (–1.114777015, 1.114777015) whereas for standard moderate variable about 63% of values fall in this range. In other words moderate curve shows relatively moderate spread of values around mean as compared to the normal curve.

## 5. Some properties of Moderate Distribution:

For X ~ M ($\mu,\delta$), following are some of the properties of this distribution which are reported without proof as they are similar to those in normal distribution.

**(i) Mode :** $\mu$ . i.e .Moderate distribution is a unimodal.

**(ii) MEDIAN:** $\mu$

**(iii) Characteristics function:** $\phi_X(t) = e^{\mu i t - \frac{\pi}{4}\delta^2 t^2}$.

**(iv) Cumulant Generating function:** $k_X(t) = \mu t + \frac{\pi}{4}\delta^2 t^2$

**(v)** $K_1 = \mu$, $K_2 = \frac{\pi}{2}\delta^2$, $K_3 = 0$, $K_4 = 0$.

i.e. All cumulants of orders higher than the second vanish identically.

**(vi)** $\beta_1 = 0, \beta_2 = 3.$

**(vii)** $\gamma_1 = 0, \gamma_2 = 0.$

**(viii)** The curve is bell-shaped and symmetrical about the line X = $\mu$.

**(ix)** Mean, median and mode of the distribution coincide.

**(x)** $\mu_{2r+1} = 0, (r = 0,1,2,\ldots).$

**(xi)** $\mu_{2r} = \pi^{r-\frac{1}{2}}.\delta^{2r}\Gamma(r+\frac{1}{2})$, where symbol $\Gamma$ represents gamma.

**(xii)** $\mu_{2r} = \frac{\pi}{2}\delta^2(2r-1)\mu_{2r-2}$

**(xiii)** $\mu_{2r} = 1.3.5..(2r-3).(2r-1).\left(\frac{\pi}{2}\right)^r \delta^{2r}, (r = 0,1,2,...).$

**(xiv)** The r[th] absolute moments about mean is considered as a measure of r[th] degree dispersion.

Here it is

$$\upsilon_r = E\left[\left|X-\mu_1'\right|^r\right] = \Gamma\left(\frac{r+1}{2}\right).\pi^{\frac{r-1}{2}}.\delta^r$$

Therefore,

$\upsilon_1 = \delta =$ M.D. about mean,

$\upsilon_2 = \frac{\pi}{2}\delta^2 =$ Variance,

$\upsilon_3 = \pi\delta^3$ ,

$\upsilon_4 = \frac{3}{4}\pi^2\delta^4 = \mu_4$,

$\upsilon_{2r-1} = (r-1)!.\pi^{r-1}.\delta^{2r-1}$,

$\upsilon_{2r} = \mu_{2r}$.

## 6. Area under Moderate Curve and related properties:

Just as the area of normal curve is measured in terms of $\sigma$ from mean $\mu$, the area of moderate distribution should be measured in terms of $\delta$ from mean $\mu$.

From the moderate table given in Appendix-A pertaining to the area under standard moderate curve, it is clear that

(i) $p(\mu-\delta < X < \mu+\delta) = 0.57506$ (4.3.1)

i.e. Mean ± 1M.D.covers 57.51% of area.

(ii) $p\left(\mu-\sqrt{\frac{\pi}{2}}\delta < X < \mu+\sqrt{\frac{\pi}{2}}\delta\right) = 0.68269$ (4.3.2)

i.e. Mean ± 1.25M.D.covers 68.27% of area.

(iii) $p(\mu-2\delta < X < \mu+2\delta) = 0.88946$ (4.3.3)

i.e. Mean ± 2M.D.covers 88.95% of area.

(iv) $p(\mu-3\delta < X < \mu+3\delta) = 0.98332$ (4.3.4)

i.e. Mean ± 3M.D.covers 98.33% of area.

(v) $p(\mu-4\delta < X < \mu+4\delta) = 0.99858$ (4.3.5)

i.e. Mean ± 4M.D.covers 99.86% of area.

In the section 3.1 and 3.2 it has been noted that there is indeed moderation in the distribution of probability among the values of moderate variable. However, there it was somewhat difficult to get the idea of extent of moderation. From the following table one can have even better idea about the extent of moderation in the distribution of probability in the moderate distribution has taken place vis-à-vis the distribution of probability in the normal distribution.

Table: 4.2 A comparison between normal distribution and moderate distribution

| **Range** | **% Frequency within Range** | | **Approximate expected frequency outside Range** | |
|---|---|---|---|---|
| | Normal distribution | Moderate distribution | Normal distribution | Moderate distribution |
| $\mu \pm$ 1D.P. | 68.2689492 $\cong$ 68 | 57.5062516 $\cong$ 57 | 3 in 10 i.e. 1 in 3 | 4 in 10 i.e. 1 in 2 |
| $\mu \pm$ 2D.P. | 95.4499736 $\cong$ 95 | 88.9459619 $\cong$ 89 | 45 in 1000 i.e. 1 in 22 | 110 in 1000 i.e. 1 in 9 |
| $\mu \pm$ 3D.P. | 99.7300204 $\cong$ 99 | 98.3318501 $\cong$ 98 | 27 in 10,000 i.e. 1 in 370 | 167 in 10,000 i.e. 1 in 60 |
| $\mu \pm$ 4D.P. | 99.9936658 $\cong$ 100 | 99.8584826 $\cong$ 100 | 6 in 1 lakh i.e. 1 in 15,787 | 141 in 1 lakh i.e. 1 in 707 |
| $\mu \pm$ 5D.P. | 99.9999427 $\cong$ 100 | 99.9933766 $\cong$ 100 | 6 in 1 crore i.e. 1 in 1,744,278 | 662 in 1 crore i.e. 1 in 15,083 |

Note: - D.P. (Dispersion parameter) for normal distribution is $\sigma$ and for moderate distribution is $\delta$.

The above table 4.2 is quite revealing about how significant moderation in the distribution of probability in normal distribution takes place when the FDD parameter $\sigma$ is replaced by another FDD parameter, $\delta$. For example, in normal distribution having $\sigma = K$, about 3 observations out of 10 are expected to lie outside the range $\mu \pm K$ whereas in moderate distribution (which is also a normal distribution) with $\delta = K$ ,about 4 observations out of 10 are expected to lie outside the range $\mu \pm K$. Similarly, for normal distribution with $\sigma = K$, about 27 observations out of 10,000 are expected to lie outside the range $\mu \pm 3K$ whereas in moderate distribution with $\delta = K$, about 167 observations out of 10,000 are expected to lie outside the range $\mu \pm 3K$.

## References:


[1]Gorard S.,(2004) : Revisiting a 90-year-old debate: the advantages of the mean deviation , The British educational research association annual conference, university of manchester,16-18 September – 2004.

[2] Hampel, F. (1997) : Is statistics too difficult?, Research Report 81, Seminar fur Statistik, Eidgenossiche Technische Hochschule, Switzerland.


## APPENDIX – A

**<u>Area under the Standard Moderate Curve and the Ordinates</u>**

**X ~ M(0,1), Mean=0, M.D.=1**

$$Y = \frac{1}{\pi}.e^{-\frac{1}{\pi}x^2}, -\infty \langle x \langle \infty \quad ; \qquad A = \int_0^x Y dY$$

| **X** | **Y** | **A** | **X** | **Y** | **A** |
|---|---|---|---|---|---|
| 0.00 | 0.318309886 | 0.000000000 | 0.41 | 0.301725451 | 0.128216228 |
| 0.01 | 0.318299754 | 0.003183065 | 0.42 | 0.300929353 | 0.131229517 |
| 0.02 | 0.318269360 | 0.006365928 | 0.43 | 0.300116249 | 0.134234759 |

| | | | | | |
|---|---|---|---|---|---|
| 0.03 | 0.318218710 | 0.009548385 | 0.44 | 0.299286288 | 0.137231785 |
| 0.04 | 0.318147813 | 0.012730234 | 0.45 | 0.298439622 | 0.140220428 |
| 0.05 | 0.318056684 | 0.015911274 | 0.46 | 0.297576407 | 0.143200523 |
| 0.06 | 0.317945339 | 0.019091301 | 0.47 | 0.296696799 | 0.146171902 |
| 0.07 | 0.317813799 | 0.022270113 | 0.48 | 0.295800960 | 0.149134405 |
| 0.08 | 0.317662090 | 0.025447509 | 0.49 | 0.294889051 | 0.152087868 |
| 0.09 | 0.317490242 | 0.028623288 | 0.50 | 0.293961240 | 0.155032133 |
| 0.10 | 0.317298285 | 0.031797247 | 0.51 | 0.293017693 | 0.157967040 |
| 0.11 | 0.317086258 | 0.034969187 | 0.52 | 0.292058580 | 0.160892435 |
| 0.12 | 0.316854200 | 0.038138905 | 0.53 | 0.291084076 | 0.163808161 |
| 0.13 | 0.316602155 | 0.041306204 | 0.54 | 0.290094354 | 0.166714065 |
| 0.14 | 0.316330173 | 0.044470882 | 0.55 | 0.289089593 | 0.169609998 |
| 0.15 | 0.316038304 | 0.047632741 | 0.56 | 0.288069973 | 0.172495808 |
| 0.16 | 0.315726603 | 0.050791582 | 0.57 | 0.287035675 | 0.175371348 |
| 0.17 | 0.315395131 | 0.053947207 | 0.58 | 0.285986883 | 0.178236473 |
| 0.18 | 0.315043950 | 0.057099419 | 0.59 | 0.284923784 | 0.181091038 |
| 0.19 | 0.314673126 | 0.060248021 | 0.60 | 0.283846567 | 0.183934901 |
| 0.20 | 0.314282731 | 0.063392816 | 0.61 | 0.282755420 | 0.186767923 |
| 0.21 | 0.313872837 | 0.066533610 | 0.62 | 0.281650537 | 0.189589964 |
| 0.22 | 0.313443523 | 0.069670208 | 0.63 | 0.280532112 | 0.192400888 |
| 0.23 | 0.312994870 | 0.072802416 | 0.64 | 0.279400340 | 0.195200562 |
| 0.24 | 0.312526962 | 0.075930042 | 0.65 | 0.278255419 | 0.197988851 |
| 0.25 | 0.312039888 | 0.079052892 | 0.66 | 0.277097549 | 0.200765627 |
| 0.26 | 0.311533739 | 0.082170776 | 0.67 | 0.275926930 | 0.203530759 |
| 0.27 | 0.311008612 | 0.085283503 | 0.68 | 0.274743765 | 0.206284123 |
| 0.28 | 0.310464604 | 0.088390885 | 0.69 | 0.273548259 | 0.209025593 |
| 0.29 | 0.309901818 | 0.091492733 | 0.70 | 0.272340616 | 0.211755047 |
| 0.30 | 0.309320360 | 0.094588859 | 0.71 | 0.271121044 | 0.214472365 |
| 0.31 | 0.308720338 | 0.097679078 | 0.72 | 0.269889751 | 0.217177430 |
| 0.32 | 0.308101865 | 0.100763204 | 0.73 | 0.268646947 | 0.219870123 |
| 0.33 | 0.307465057 | 0.103841054 | 0.74 | 0.267392843 | 0.222550332 |
| 0.34 | 0.306810033 | 0.106912445 | 0.75 | 0.266127650 | 0.225217943 |
| 0.35 | 0.306136913 | 0.109977195 | 0.76 | 0.264851582 | 0.227872848 |
| 0.36 | 0.305445825 | 0.113035123 | 0.77 | 0.263564854 | 0.230514939 |
| 0.37 | 0.304736896 | 0.116086052 | 0.78 | 0.262267679 | 0.233144111 |
| 0.38 | 0.304010258 | 0.119129802 | 0.79 | 0.260960275 | 0.235760259 |
| 0.39 | 0.303266046 | 0.122166198 | 0.80 | 0.259642858 | 0.238363283 |
| 0.40 | 0.302504396 | 0.125195065 | 0.81 | 0.258315647 | 0.240953083 |

**Area under the Standard Moderate Curve and the Ordinates**

| X | Y | A | X | Y | A |
|---|---|---|---|---|---|
| 0.82 | 0.256978860 | 0.243529564 | 1.32 | 0.182801753 | 0.353876962 |
| 0.83 | 0.255632716 | 0.246092629 | 1.33 | 0.181266266 | 0.355697314 |
| 0.84 | 0.254277436 | 0.248642187 | 1.34 | 0.179732235 | 0.357502288 |
| 0.85 | 0.252913239 | 0.251178148 | 1.35 | 0.178199841 | 0.359291960 |
| 0.86 | 0.251540347 | 0.253700423 | 1.36 | 0.176669265 | 0.361066316 |
| 0.87 | 0.250158981 | 0.256208927 | 1.37 | 0.175140685 | 0.362825346 |
| 0.88 | 0.248769364 | 0.258703575 | 1.38 | 0.173614278 | 0.364569131 |
| 0.89 | 0.247371717 | 0.261184287 | 1.39 | 0.172090218 | 0.366297663 |
| 0.90 | 0.245966263 | 0.263650983 | 1.40 | 0.170568677 | 0.368010936 |
| 0.91 | 0.244553225 | 0.266103586 | 1.41 | 0.169049827 | 0.369709038 |
| 0.92 | 0.243132826 | 0.268542023 | 1.42 | 0.167533836 | 0.371391964 |
| 0.93 | 0.241705290 | 0.270966219 | 1.43 | 0.166020871 | 0.373059745 |
| 0.94 | 0.240270838 | 0.273376105 | 1.44 | 0.164511095 | 0.374712384 |

| | | | | | |
|---|---|---|---|---|---|
| 0.95 | 0.238829694 | 0.275771613 | 1.45 | 0.163004671 | 0.376349970 |
| 0.96 | 0.237382082 | 0.278152676 | 1.46 | 0.161501760 | 0.377972508 |
| 0.97 | 0.235928223 | 0.280519233 | 1.47 | 0.160002519 | 0.379580034 |
| 0.98 | 0.234468342 | 0.282871223 | 1.48 | 0.158507105 | 0.381172562 |
| 0.99 | 0.233002661 | 0.285208583 | 1.49 | 0.157015671 | 0.382750181 |
| 1.00 | 0.231531401 | 0.287531258 | 1.50 | 0.155528368 | 0.384312906 |
| 1.01 | 0.230054785 | 0.289839193 | 1.51 | 0.154045347 | 0.385860778 |
| 1.02 | 0.228573035 | 0.292132336 | 1.52 | 0.152566754 | 0.387393818 |
| 1.03 | 0.227086371 | 0.294410637 | 1.53 | 0.151092734 | 0.388912119 |
| 1.04 | 0.225595014 | 0.296674048 | 1.54 | 0.149623429 | 0.390415703 |
| 1.05 | 0.224099185 | 0.298922523 | 1.55 | 0.148158981 | 0.391904618 |
| 1.06 | 0.222599102 | 0.301156018 | 1.56 | 0.146699526 | 0.393378912 |
| 1.07 | 0.221094985 | 0.303374491 | 1.57 | 0.145245201 | 0.394838616 |
| 1.08 | 0.219587051 | 0.305577904 | 1.58 | 0.143796139 | 0.396283824 |
| 1.09 | 0.218075518 | 0.307766220 | 1.59 | 0.142352471 | 0.397714569 |
| 1.10 | 0.216560603 | 0.309939403 | 1.60 | 0.140914325 | 0.399130903 |
| 1.11 | 0.215042521 | 0.312097421 | 1.61 | 0.139481829 | 0.400532884 |
| 1.12 | 0.213521487 | 0.314240243 | 1.62 | 0.138055106 | 0.401920550 |
| 1.13 | 0.211997716 | 0.316367841 | 1.63 | 0.136634279 | 0.403293997 |
| 1.14 | 0.210471418 | 0.318480189 | 1.64 | 0.135219465 | 0.404653266 |
| 1.15 | 0.208942808 | 0.320577262 | 1.65 | 0.133810782 | 0.405998417 |
| 1.16 | 0.207412095 | 0.322659038 | 1.66 | 0.132408345 | 0.407329511 |
| 1.17 | 0.205879488 | 0.324725497 | 1.67 | 0.131012266 | 0.408646612 |
| 1.18 | 0.204345197 | 0.326776621 | 1.68 | 0.129622654 | 0.409949769 |
| 1.19 | 0.202809429 | 0.328812395 | 1.69 | 0.128239618 | 0.411239079 |
| 1.20 | 0.201272388 | 0.330832805 | 1.70 | 0.126863261 | 0.412514592 |
| 1.21 | 0.199734281 | 0.332837838 | 1.71 | 0.125493687 | 0.413776375 |
| 1.22 | 0.19819531 | 0.334827486 | 1.72 | 0.124130996 | 0.415024496 |
| 1.23 | 0.196655677 | 0.336801741 | 1.73 | 0.122775285 | 0.416259024 |
| 1.24 | 0.195115582 | 0.338760603 | 1.74 | 0.121426651 | 0.417480016 |
| 1.25 | 0.193575225 | 0.340704057 | 1.75 | 0.120085185 | 0.418687573 |
| 1.26 | 0.192034802 | 0.342631967 | 1.76 | 0.118750979 | 0.419881751 |
| 1.27 | 0.190494510 | 0.344544630 | 1.77 | 0.117424122 | 0.421062624 |
| 1.28 | 0.188954543 | 0.346441857 | 1.78 | 0.116104698 | 0.422230264 |
| 1.29 | 0.187415094 | 0.348323721 | 1.79 | 0.114792791 | 0.423384748 |
| 1.30 | 0.185876353 | 0.350190192 | 1.80 | 0.113488483 | 0.424526137 |
| 1.31 | 0.184338510 | 0.352041247 | 1.81 | 0.112191853 | 0.425654535 |

**Area under the Standard Moderate Curve and the Ordinates**

| X | Y | A | X | Y | A |
|---|---|---|---|---|---|
| 1.82 | 0.110902976 | 0.426770006 | 2.32 | 0.057383354 | 0.467921847 |
| 1.83 | 0.109621927 | 0.427872627 | 2.33 | 0.056540254 | 0.468491458 |
| 1.84 | 0.108348777 | 0.428962476 | 2.34 | 0.055705996 | 0.469052682 |
| 1.85 | 0.107083597 | 0.430039633 | 2.35 | 0.054880552 | 0.469605608 |
| 1.86 | 0.105826452 | 0.431104179 | 2.36 | 0.054063898 | 0.470150324 |
| 1.87 | 0.104577409 | 0.432156193 | 2.37 | 0.053256006 | 0.470686916 |
| 1.88 | 0.103336529 | 0.433195747 | 2.38 | 0.052456847 | 0.471215474 |
| 1.89 | 0.102103872 | 0.434222944 | 2.39 | 0.051666391 | 0.471736083 |
| 1.90 | 0.100879497 | 0.435237857 | 2.40 | 0.050884606 | 0.472248831 |
| 1.91 | 0.099663459 | 0.436240566 | 2.41 | 0.050111461 | 0.472753804 |
| 1.92 | 0.098455811 | 0.437231158 | 2.42 | 0.049346921 | 0.473251089 |
| 1.93 | 0.097256605 | 0.438209714 | 2.43 | 0.048590952 | 0.473740769 |
| 1.94 | 0.096065890 | 0.439176321 | 2.44 | 0.047843519 | 0.474222934 |
| 1.95 | 0.094883711 | 0.440131063 | 2.45 | 0.047104583 | 0.474697668 |
| 1.96 | 0.093710115 | 0.441074017 | 2.46 | 0.046374108 | 0.475165055 |

| | | | | | |
|---|---|---|---|---|---|
| 1.97 | 0.092545143 | 0.442005288 | 2.47 | 0.045652054 | 0.475625179 |
| 1.98 | 0.091388835 | 0.442924953 | 2.48 | 0.044938382 | 0.476078125 |
| 1.99 | 0.090241230 | 0.443833097 | 2.49 | 0.044233051 | 0.476523975 |
| 2.00 | 0.089102362 | 0.444729809 | 2.50 | 0.043536018 | 0.476962814 |
| 2.01 | 0.087972267 | 0.445615177 | 2.51 | 0.042847242 | 0.477394724 |
| 2.02 | 0.086850976 | 0.446489287 | 2.52 | 0.042166678 | 0.477819787 |
| 2.03 | 0.085738519 | 0.447352228 | 2.53 | 0.041494282 | 0.478238085 |
| 2.04 | 0.084634922 | 0.448204089 | 2.54 | 0.040830009 | 0.478649700 |
| 2.05 | 0.083540212 | 0.449044950 | 2.55 | 0.040173813 | 0.479054712 |
| 2.06 | 0.082454412 | 0.449874917 | 2.56 | 0.039525646 | 0.479453203 |
| 2.07 | 0.081377544 | 0.450694071 | 2.57 | 0.038885461 | 0.479845252 |
| 2.08 | 0.080309627 | 0.451502501 | 2.58 | 0.038253210 | 0.480230939 |
| 2.09 | 0.079250679 | 0.452300296 | 2.59 | 0.037628843 | 0.480610343 |
| 2.10 | 0.078200715 | 0.453087546 | 2.60 | 0.037012310 | 0.480983540 |
| 2.11 | 0.077159750 | 0.453864342 | 2.61 | 0.036403562 | 0.481350613 |
| 2.12 | 0.076127795 | 0.454630773 | 2.62 | 0.035802546 | 0.481711638 |
| 2.13 | 0.075104860 | 0.455386930 | 2.63 | 0.035209212 | 0.482066690 |
| 2.14 | 0.074090953 | 0.456132902 | 2.64 | 0.034623506 | 0.482415848 |
| 2.15 | 0.073086081 | 0.456868780 | 2.65 | 0.034045376 | 0.482759186 |
| 2.16 | 0.072090249 | 0.457594649 | 2.66 | 0.033474768 | 0.483096781 |
| 2.17 | 0.071103458 | 0.458310611 | 2.67 | 0.032911629 | 0.483428707 |
| 2.18 | 0.070125710 | 0.459016750 | 2.68 | 0.032355903 | 0.483755039 |
| 2.19 | 0.069157005 | 0.459713157 | 2.69 | 0.031807535 | 0.484075850 |
| 2.20 | 0.068197339 | 0.460399922 | 2.70 | 0.031266471 | 0.484391214 |
| 2.21 | 0.067246709 | 0.461077136 | 2.71 | 0.030732655 | 0.484701204 |
| 2.22 | 0.066305109 | 0.461744888 | 2.72 | 0.030206029 | 0.485005891 |
| 2.23 | 0.065372531 | 0.462403269 | 2.73 | 0.029686537 | 0.485305348 |
| 2.24 | 0.064448967 | 0.463052370 | 2.74 | 0.029174122 | 0.485599646 |
| 2.25 | 0.063534407 | 0.463692280 | 2.75 | 0.028668727 | 0.485888854 |
| 2.26 | 0.062628836 | 0.464323089 | 2.76 | 0.028170294 | 0.486173044 |
| 2.27 | 0.061732244 | 0.464944887 | 2.77 | 0.027678764 | 0.486452283 |
| 2.28 | 0.060844613 | 0.465557760 | 2.78 | 0.027194079 | 0.486726642 |
| 2.29 | 0.059965927 | 0.466161806 | 2.79 | 0.026716181 | 0.486996188 |
| 2.30 | 0.059096169 | 0.466757110 | 2.80 | 0.026245010 | 0.487260988 |
| 2.31 | 0.058235318 | 0.467343760 | 2.81 | 0.025780508 | 0.487521109 |

**Area under the Standard Moderate Curve and the Ordinates**

| X | Y | A | X | Y | A |
|---|---|---|---|---|---|
| 2.82 | 0.025322615 | 0.487776619 | 3.18 | 0.012732382 | 0.494414008 |
| 2.83 | 0.024871271 | 0.488027583 | 3.19 | 0.012476815 | 0.494540051 |
| 2.84 | 0.024426416 | 0.488274067 | 3.20 | 0.012225601 | 0.494663559 |
| 2.85 | 0.023987991 | 0.488516133 | 3.21 | 0.011978681 | 0.494784577 |
| 2.86 | 0.023555936 | 0.488753848 | 3.22 | 0.011736002 | 0.494903147 |
| 2.87 | 0.023130190 | 0.488987273 | 3.23 | 0.011497506 | 0.495019311 |
| 2.88 | 0.022710693 | 0.489216473 | 3.24 | 0.011263141 | 0.495133111 |
| 2.89 | 0.022297385 | 0.489441508 | 3.25 | 0.011032850 | 0.495244587 |
| 2.90 | 0.021890204 | 0.489662441 | 3.26 | 0.010806580 | 0.495353781 |
| 2.91 | 0.021489092 | 0.489879332 | 3.27 | 0.010584277 | 0.495460732 |
| 2.92 | 0.021093986 | 0.490092243 | 3.28 | 0.010365887 | 0.495565480 |
| 2.93 | 0.020704827 | 0.490301232 | 3.29 | 0.010151357 | 0.495668063 |
| 2.94 | 0.020321553 | 0.490506359 | 3.30 | 0.009940633 | 0.495768520 |
| 2.95 | 0.019944105 | 0.490707683 | 3.31 | 0.009733665 | 0.495866888 |
| 2.96 | 0.019572421 | 0.490905261 | 3.32 | 0.009530398 | 0.495963205 |
| 2.97 | 0.019206442 | 0.491099150 | 3.33 | 0.009330783 | 0.496057508 |
| 2.98 | 0.018846105 | 0.491289408 | 3.34 | 0.009134766 | 0.496149833 |

| | | | | | |
|---|---|---|---|---|---|
| 2.99 | 0.018491352 | 0.491476091 | 3.35 | 0.008942299 | 0.496240216 |
| 3.00 | 0.018142122 | 0.491659254 | 3.36 | 0.008753329 | 0.496328691 |
| 3.01 | 0.017798354 | 0.491838952 | 3.37 | 0.008567807 | 0.496415294 |
| 3.02 | 0.017459989 | 0.492015239 | 3.38 | 0.008385684 | 0.496500058 |
| 3.03 | 0.017126966 | 0.492188169 | 3.39 | 0.008206909 | 0.496583018 |
| 3.04 | 0.016799225 | 0.492357796 | 3.40 | 0.008031434 | 0.496664207 |
| 3.05 | 0.016476707 | 0.492524171 | 3.41 | 0.007859211 | 0.496743658 |
| 3.06 | 0.016159352 | 0.492687347 | 3.42 | 0.007690191 | 0.496821402 |
| 3.07 | 0.015847100 | 0.492847375 | 3.43 | 0.007524327 | 0.496897472 |
| 3.08 | 0.015539893 | 0.493004306 | 3.44 | 0.007361572 | 0.496971899 |
| 3.09 | 0.015237672 | 0.493158189 | 3.45 | 0.007201879 | 0.497044714 |
| 3.10 | 0.014940377 | 0.493309076 | 3.46 | 0.007045202 | 0.497115947 |
| 3.11 | 0.014647949 | 0.493457013 | 3.47 | 0.006891494 | 0.497185628 |
| 3.12 | 0.014360331 | 0.493602051 | 3.48 | 0.006740711 | 0.497253786 |
| 3.13 | 0.014077465 | 0.493744236 | 3.49 | 0.006592807 | 0.497320452 |
| 3.14 | 0.013799291 | 0.493883616 | 3.50 | 0.006447737 | 0.497385652 |
| 3.01 | 0.017798354 | 0.491838952 | 3.51 | 0.006305459 | 0.497449416 |
| 3.02 | 0.017459989 | 0.492015239 | 3.52 | 0.006165928 | 0.497511770 |
| 3.03 | 0.017126966 | 0.492188169 | 3.53 | 0.006029100 | 0.497572743 |
| 3.04 | 0.016799225 | 0.492357796 | 3.54 | 0.005894933 | 0.497632361 |
| 3.05 | 0.016476707 | 0.492524171 | 3.55 | 0.005763385 | 0.497690651 |
| 3.06 | 0.016159352 | 0.492687347 | 3.56 | 0.005634414 | 0.497747638 |
| 3.07 | 0.015847100 | 0.492847375 | 3.57 | 0.005507979 | 0.497803347 |
| 3.08 | 0.015539893 | 0.493004306 | 3.58 | 0.005384038 | 0.497857805 |
| 3.09 | 0.015237672 | 0.493158189 | 3.59 | 0.005262550 | 0.497911036 |
| 3.10 | 0.014940377 | 0.493309076 | 3.60 | 0.005143477 | 0.497963065 |
| 3.11 | 0.014647949 | 0.493457013 | 3.61 | 0.005026778 | 0.498013914 |
| 3.12 | 0.014360331 | 0.493602051 | 3.62 | 0.004912413 | 0.498063608 |
| 3.13 | 0.014077465 | 0.493744236 | 3.63 | 0.004800345 | 0.498112170 |
| 3.14 | 0.013799291 | 0.493883616 | 3.64 | 0.004690535 | 0.498159622 |
| 3.15 | 0.013525754 | 0.494020237 | 3.65 | 0.004582946 | 0.498205988 |
| 3.16 | 0.013256794 | 0.494154146 | 3.66 | 0.004477539 | 0.498251289 |
| 3.17 | 0.012992356 | 0.494285388 | 3.67 | 0.004374278 | 0.498295546 |

**Area under the Standard Moderate Curve and the Ordinates**

| X | Y | A | X | Y | A |
|---|---|---|---|---|---|
| 3.68 | 0.004273126 | 0.498338781 | 4.19 | 0.001190943 | 0.499585814 |
| 3.69 | 0.004174047 | 0.498381015 | 4.20 | 0.001159558 | 0.499597566 |
| 3.70 | 0.004077007 | 0.498422269 | 4.21 | 0.001128929 | 0.499609008 |
| 3.71 | 0.003981968 | 0.498462562 | 4.22 | 0.001099038 | 0.499620147 |
| 3.72 | 0.003888898 | 0.498501915 | 4.23 | 0.001069871 | 0.499630991 |
| 3.73 | 0.003797761 | 0.498540347 | 4.24 | 0.001041412 | 0.499641547 |
| 3.74 | 0.003708524 | 0.498577876 | 4.25 | 0.001013645 | 0.499651821 |
| 3.75 | 0.003621153 | 0.498614523 | 4.26 | 0.000986556 | 0.499661822 |
| 3.76 | 0.003535616 | 0.498650306 | 4.27 | 0.000960130 | 0.499671555 |
| 3.77 | 0.003451879 | 0.498685242 | 4.28 | 0.000934352 | 0.499681027 |
| 3.78 | 0.003369911 | 0.498719349 | 4.29 | 0.000909208 | 0.499690244 |
| 3.79 | 0.003289680 | 0.498752646 | 4.30 | 0.000884684 | 0.499699213 |
| 3.80 | 0.003211155 | 0.498785148 | 4.31 | 0.000860767 | 0.499707940 |
| 3.81 | 0.003134304 | 0.498816874 | 4.32 | 0.000837444 | 0.499716430 |
| 3.82 | 0.003059098 | 0.498847840 | 4.33 | 0.000814700 | 0.499724690 |
| 3.83 | 0.002985507 | 0.498878062 | 4.34 | 0.000792524 | 0.499732726 |
| 3.84 | 0.002913500 | 0.498907555 | 4.35 | 0.000770902 | 0.499740543 |
| 3.85 | 0.002843049 | 0.498936337 | 4.36 | 0.000749823 | 0.499748146 |
| 3.86 | 0.002774125 | 0.498964421 | 4.37 | 0.000729273 | 0.499755541 |

| | | | | | |
|---|---|---|---|---|---|
| 3.87 | 0.002706700 | 0.498991824 | 4.38 | 0.000709242 | 0.499762733 |
| 3.88 | 0.002640745 | 0.499018560 | 4.39 | 0.000689717 | 0.499769727 |
| 3.89 | 0.002576233 | 0.499044644 | 4.40 | 0.000670686 | 0.499776529 |
| 3.90 | 0.002513138 | 0.499070090 | 4.41 | 0.000652139 | 0.499783143 |
| 3.91 | 0.002451431 | 0.499094911 | 4.42 | 0.000634065 | 0.499789573 |
| 3.92 | 0.002391088 | 0.499119123 | 4.43 | 0.000616452 | 0.499795826 |
| 3.93 | 0.002332081 | 0.499142738 | 4.44 | 0.000599291 | 0.499801904 |
| 3.94 | 0.002274386 | 0.499165769 | 4.45 | 0.000582570 | 0.499807813 |
| 3.95 | 0.002217977 | 0.499188230 | 4.46 | 0.000566279 | 0.499813557 |
| 3.96 | 0.002162829 | 0.499210133 | 4.47 | 0.000550410 | 0.499819140 |
| 3.97 | 0.002108918 | 0.499231490 | 4.48 | 0.000534950 | 0.499824566 |
| 3.98 | 0.002056220 | 0.499252315 | 4.49 | 0.000519892 | 0.499829840 |
| 3.99 | 0.002004712 | 0.499272619 | 4.50 | 0.000505226 | 0.499834966 |
| 4.00 | 0.001954369 | 0.499292413 | 4.51 | 0.000490942 | 0.499839946 |
| 4.01 | 0.001905169 | 0.499311710 | 4.52 | 0.000477031 | 0.499844786 |
| 4.02 | 0.001857089 | 0.499330520 | 4.53 | 0.000463486 | 0.499849488 |
| 4.03 | 0.001810108 | 0.499348855 | 4.54 | 0.000450296 | 0.499854057 |
| 4.04 | 0.001764203 | 0.499366726 | 4.55 | 0.000437453 | 0.499858495 |
| 4.05 | 0.001719352 | 0.499384143 | 4.56 | 0.000424950 | 0.499862807 |
| 4.06 | 0.001675535 | 0.499401116 | 4.57 | 0.000412778 | 0.499866995 |
| 4.07 | 0.001632731 | 0.499417657 | 4.58 | 0.000400929 | 0.499871063 |
| 4.08 | 0.001590919 | 0.499433774 | 4.59 | 0.000389396 | 0.499875015 |
| 4.09 | 0.001550079 | 0.499449479 | 4.60 | 0.000378170 | 0.499878852 |
| 4.1 | 0.001510191 | 0.499464779 | 4.61 | 0.000367244 | 0.499882579 |
| 4.11 | 0.001471236 | 0.499479685 | 4.62 | 0.000356612 | 0.499886198 |
| 4.12 | 0.001433195 | 0.499494207 | 4.63 | 0.000346265 | 0.499889712 |
| 4.13 | 0.001396048 | 0.499508352 | 4.64 | 0.000336197 | 0.499893124 |
| 4.14 | 0.001359778 | 0.499522131 | 4.65 | 0.000326401 | 0.499896437 |
| 4.15 | 0.001324365 | 0.499535551 | 4.66 | 0.000316870 | 0.499899653 |
| 4.16 | 0.001289793 | 0.499548621 | 4.67 | 0.000307598 | 0.499902775 |
| 4.17 | 0.001256043 | 0.499561349 | 4.68 | 0.000298578 | 0.499905806 |
| 4.18 | 0.001223099 | 0.499573744 | 4.69 | 0.000289804 | 0.499908748 |

**Area under the Standard Moderate Curve and the Ordinates**

| X | Y | A | X | Y | A |
|---|---|---|---|---|---|
| 4.70 | 0.000281270 | 0.499911603 | 5.21 | 0.000056292 | 0.499983876 |
| 4.71 | 0.000272970 | 0.499914374 | 5.22 | 0.000054453 | 0.499984429 |
| 4.72 | 0.000264898 | 0.499917063 | 5.23 | 0.000052672 | 0.499984965 |
| 4.73 | 0.000257049 | 0.499919673 | 5.24 | 0.000050945 | 0.499985483 |
| 4.74 | 0.000249416 | 0.499922205 | 5.25 | 0.000049272 | 0.499985984 |
| 4.75 | 0.000241994 | 0.499924662 | 5.26 | 0.000047651 | 0.499986469 |
| 4.76 | 0.000234779 | 0.499927045 | 5.27 | 0.000046081 | 0.499986937 |
| 4.77 | 0.000227763 | 0.499929358 | 5.28 | 0.000044559 | 0.499987390 |
| 4.78 | 0.000220944 | 0.499931601 | 5.29 | 0.000043085 | 0.499987829 |
| 4.79 | 0.000214315 | 0.499933777 | 5.30 | 0.000041656 | 0.499988252 |
| 4.80 | 0.000207872 | 0.499935888 | 5.31 | 0.000040273 | 0.499988662 |
| 4.81 | 0.000201609 | 0.499937935 | 5.32 | 0.000038933 | 0.499989058 |
| 4.82 | 0.000195523 | 0.499939921 | 5.33 | 0.000037635 | 0.499989441 |
| 4.83 | 0.000189608 | 0.499941847 | 5.34 | 0.000036379 | 0.499989811 |
| 4.84 | 0.000183861 | 0.499943714 | 5.35 | 0.000035162 | 0.499990168 |
| 4.85 | 0.000178277 | 0.499945524 | 5.36 | 0.000033983 | 0.499990514 |
| 4.86 | 0.000172851 | 0.499947280 | 5.37 | 0.000032842 | 0.499990848 |
| 4.87 | 0.000167579 | 0.499948982 | 5.38 | 0.000031737 | 0.499991171 |
| 4.88 | 0.000162458 | 0.499950632 | 5.39 | 0.000030668 | 0.499991483 |
| 4.89 | 0.000157484 | 0.499952231 | 5.40 | 0.000029632 | 0.499991785 |

| | | | | | |
|---|---|---|---|---|---|
| 4.90 | 0.000152652 | 0.499953782 | 5.41 | 0.000028630 | 0.499992076 |
| 4.91 | 0.000147959 | 0.499955285 | 5.42 | 0.000027660 | 0.499992357 |
| 4.92 | 0.000143401 | 0.499956742 | 5.43 | 0.000026721 | 0.499992629 |
| 4.93 | 0.000138975 | 0.499958153 | 5.44 | 0.000025812 | 0.499992892 |
| 4.94 | 0.000134676 | 0.499959522 | 5.45 | 0.000024933 | 0.499993145 |
| 4.95 | 0.000130503 | 0.499960847 | 5.46 | 0.000024082 | 0.499993391 |
| 4.96 | 0.000126450 | 0.499962132 | 5.47 | 0.000023258 | 0.499993627 |
| 4.97 | 0.000122516 | 0.499963377 | 5.48 | 0.000022462 | 0.499993856 |
| 4.98 | 0.000118696 | 0.499964583 | 5.49 | 0.000021691 | 0.499994077 |
| 4.99 | 0.000114989 | 0.499965751 | 5.50 | 0.000020945 | 0.499994290 |
| 5.00 | 0.000111390 | 0.499966883 | 5.51 | 0.000020224 | 0.499994496 |
| 5.01 | 0.000107896 | 0.499967979 | 5.52 | 0.000019526 | 0.499994694 |
| 5.02 | 0.000104506 | 0.499969041 | 5.53 | 0.000018851 | 0.499994886 |
| 5.03 | 0.000101216 | 0.499970070 | 5.54 | 0.000018199 | 0.499995071 |
| 5.04 | 0.000098023 | 0.499971066 | 5.55 | 0.000017567 | 0.499995250 |
| 5.05 | 0.000094925 | 0.499972030 | 5.56 | 0.000016957 | 0.499995423 |
| 5.06 | 0.000091918 | 0.499972965 | 5.57 | 0.000016367 | 0.499995589 |
| 5.07 | 0.000089002 | 0.499973869 | 5.58 | 0.000015796 | 0.499995750 |
| 5.08 | 0.000086172 | 0.499974745 | 5.59 | 0.000015244 | 0.499995905 |
| 5.09 | 0.000083427 | 0.499975593 | 5.60 | 0.000014711 | 0.499996055 |
| 5.10 | 0.000080765 | 0.499976414 | 5.61 | 0.000014195 | 0.499996200 |
| 5.11 | 0.000078182 | 0.499977208 | 5.62 | 0.000013697 | 0.499996339 |
| 5.12 | 0.000075677 | 0.499977978 | 5.63 | 0.000013215 | 0.499996474 |
| 5.13 | 0.000073248 | 0.499978722 | 5.64 | 0.000012749 | 0.499996603 |
| 5.14 | 0.000070892 | 0.499979443 | 5.65 | 0.000012299 | 0.499996729 |
| 5.15 | 0.000068608 | 0.499980140 | 5.66 | 0.000011864 | 0.499996849 |
| 5.16 | 0.000066393 | 0.499980815 | 5.67 | 0.000011444 | 0.499996966 |
| 5.17 | 0.000064245 | 0.499981468 | 5.68 | 0.000011038 | 0.499997078 |
| 5.18 | 0.000062163 | 0.499982100 | 5.69 | 0.000010646 | 0.499997187 |
| 5.19 | 0.000060145 | 0.499982712 | 5.70 | 0.000010267 | 0.499997291 |
| 5.20 | 0.000058188 | 0.499983303 | 5.71 | 0.000009900 | 0.499997392 |

**Area under the Standard Moderate Curve and the Ordinates**

| X | Y | A | X | Y | A |
|---|---|---|---|---|---|
| 5.72 | 0.000009547 | 0.499997489 | 5.86 | 0.000005698 | 0.499998535 |
| 5.73 | 0.000009205 | 0.499997583 | 5.87 | 0.000005489 | 0.499998590 |
| 5.74 | 0.000008875 | 0.499997674 | 5.88 | 0.000005288 | 0.499998644 |
| 5.75 | 0.000008556 | 0.499997761 | 5.89 | 0.000005093 | 0.499998696 |
| 5.76 | 0.000008248 | 0.499997845 | 5.90 | 0.000004906 | 0.499998746 |
| 5.77 | 0.000007951 | 0.499997926 | 5.91 | 0.000004725 | 0.499998794 |
| 5.78 | 0.000007664 | 0.499998004 | 5.92 | 0.000004550 | 0.499998841 |
| 5.79 | 0.000007387 | 0.499998079 | 5.93 | 0.000004382 | 0.499998885 |
| 5.80 | 0.000007119 | 0.499998152 | 5.94 | 0.000004219 | 0.499998928 |
| 5.81 | 0.000006861 | 0.499998221 | 5.95 | 0.000004063 | 0.499998970 |
| 5.82 | 0.000006612 | 0.499998289 | 5.96 | 0.000003911 | 0.499999010 |
| 5.83 | 0.000006371 | 0.499998354 | 5.97 | 0.000003766 | 0.499999048 |
| 5.84 | 0.000006139 | 0.499998416 | 5.98 | 0.000003625 | 0.499999085 |
| 5.85 | 0.000005915 | 0.499998476 | 5.99 | 0.000003490 | 0.499999121 |