

**CURRICULUM FOR M.PHIL & PH. D. (MATHEMATICS)** (COURSE WORK)

|         |  |           |
|---------|--|-----------|
| Paper-1 | Research Methodology                               | (32) Hrs. |
| Paper-2 | Fundamental of Mathematics (Select any four units) | (32) Hrs. |
| Paper-3 | Special paper (Select any one)                     | (32) Hrs. |

| <b>Paper - 1</b> | <b>Research Methodology</b>   | <b>Hrs.</b> |
|------------------|---|-------------|
| Unit-I           | Mathematical Software   | (8)         |
| Unit-II          | Numerical Analysis : Interpretation, Integration & Differentiation, Numerical of deferential equation | (8)         |
| Unit-III         | Dimensional Analysis / Special Functions / Operation Research   | (8)         |
| Unit-IV          | Integral Transform, Hankel Transform  | (8)         |

| <b>Paper - 2</b> | <b>Foundation of Mathematics (Select any four Units)</b> | <b>Hrs.</b> |
|------------------|--|-------------|
| Unit-I           | Complex Analysis   | (8)         |
| Unit-II          | Abstract Algebra   | (8)         |
| Unit-III         | Linear Algebra   | (8)         |
| Unit- IV         | Ordinary Differential Equations                          | (8)         |
| Unit-V           | Difference Equations                                     | (8)         |
| Unit-VI          | Real Analysis  | (8)         |
| Unit-VII         | Partial Differential Equations                           | (8)         |
| Unit-VIII        | Laplace Transform Fourier Transform                      | (8)         |

| <b>Paper - 3</b> | <b>Special Paper (Select any one of the following papers)</b> | <b>Hrs.</b> |
|------------------|---|-------------|
| I                | Methods of weighted Residules and collocation method          | (32)        |
| II               | Symmetries of Differential Equations                          | (32)        |
| III              | Numerical solution of Boundary value problems and FEM         | (32)        |
| IV               | Elements Number Theory  | (32)        |

Each paper will consist of 4 units. And each unit will be distributed in 8 hours Total hours of content course will be of 96 hours.

Subject: Mathematics Syllabus for Paper-I (Research Methodology)  
(Course work of PhD)

**UNIT-I MATHEMATICAL SOFTWARE**

Introduction to Matlab, variable and array, subarrays, displaying output data, data files operation on array, hierarchy of operation on array, built in function in Matlab

Introduction to plotting, graph window, two dimensional plot, multiple plot, components of graph(legend, title, graphical image, comment, 3D graph, additional plotting features  
Subplots, polar plots.

**UNIT-II NUMERICAL ANALYSIS**

**Numerical Differentiations:**

Numerical differentiations, Optimum choice of step-length, Extrapolation methods, Partial differentiations.

**Numerical Integrations:**

Numerical Integrations, Methods based on interpolations, Methods based on Undetermined coefficients, Composite integration methods, Romberg Integration.

**UNIT-III**

**Operation Research**

Introduction to Various Types of Inventory Problems, Method with known Demand Function, Economic Order Quantity (EOQ), Deterministic Inventory Problems when Shortages are Allowed Deterministic Inventory Problems when Shortages are Not Allowed, EOQ Problems with Price Breaks

**Special Functions**

The Euler or Mascheroni constant  $\gamma$ , The Gamma function, A series for  $\Gamma(z)/\Gamma(z)$ , Evaluation of  $\Gamma(1)$  and  $\Gamma(1)'$ , The Euler product for  $\Gamma(z)$ , The difference equation  $\Gamma(z+1) = z\Gamma(z)$ . The order symbols  $o$  and  $0$ , Evaluation of certain infinite products, Euler integral for  $\Gamma(z)$ ,

The Beta function, The value of  $\Gamma(z)\Gamma(1-z)$ , The factorial function, Legendre's duplication formula, Gauss' multiplication theorem, A summation formula due to Euler, The behaviour of  $\log(z)$  for large  $z$ .

**UNIT-IV**

**HUNKEL TRANSFORM**

Introduction and definition of Hankel transform, Examples on Hankel Transform, Operational properties of the Hankel transform, Examples of Hunkel Transform

Application of Hankel transforms to partial differential equations, Applications of finite Hankel transforms.

**Reference Books :**

1. Lokenath Debnath: Integral Transform and their applications, CRC Pub., 1995.
2. Kantiswarup, P.K.Gupta and Manmohan: Operations Research, Sultan chand and Sons.
3. E. D. Rainville, Special Functions, McMillan, New York, 1990

4. Chapman Stephen: Matlab programming for engineers, Thompson learning, 2004
5. M.K. Jain, S.R.K. Iyenger, R.K. Jain: "Numerical Methods for scientific and engineering computations", VI – edition, New Age International Publishers

**Subject: Mathematics Syllabus for Paper-II  
(Course work of PhD)**

**Unit-1: Real Analysis**

**1. Elements of set theory**

Countable and uncountable sets, Infinite sets, Cardinal numbers and their arithmetic, Schroder-Bernstein theorem. Cantor's theorem and the continuum hypothesis, Poset, Axiom of choice, Housdorff maximality principle, Tukey's theorem, Zorn's lemma, well ordering theorem.

**2. Sequence and series of real, numbers**

Sequence, Limit of a sequence, Convergence of sequence, limit superior, limit inferior of a sequence of real numbers, Cauchy sequence, Series, Convergence of Series, tests for Convergence of Series.

**3. Riemann Integral**

Darboux's upper and lower sums, lower and upper integrals, Riemann integral criterion for Riemann integrability, Properties of integral functions, Fundamental theorem of calculus, the integral as a limit of sum.

**4. Improper Integrals.**

Improper integrals, types of improper integrals, Convergence of improper integrals, test integrals and some tests for convergence. Gamma and Beta functions,

**Unit-2: Complex Analysis**

1. Complex functions as mappings. Analytic function, Bilinear transformation, Conformal mappings,
2. Power series representation of Analytic functions, Radius of convergence, Cauchy's estimate, Zero's of analytic functions, Integration in complex plane. Fundamental theorem of algebra. Maximum modulus theorem,
3. Singularities, Classification, Poles and essential singularities, Singular point, Laurent series, Residues and evaluation of integrals.
4. Entire functions, Weierstrass theorem,

**Unit-3: Algebra**

1. Groups, subgroups, normal subgroups, quotient groups, homeomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.
2. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.
3. Polynomial rings and irreducibility criteria.

4. Fields, finite fields, field extensions, Galois Theory.

#### **Unit-4: Linear Algebra**

1. Vector Spaces, Subspaces, Linear dependence and Linear independence of Vectors, System of linear equations. Basis and Dimension of a Vector Space, Quotient Space.
- 2, Linear transformations, Range and Null Space of a linear transformation, Trace of a matrix, Eigen values and Eigen vectors, The Cayley-Hamilton theorem, Minimal polynomial,
- 3, Modules, Sub-modules, Free modules. Faithful modules. Direct sum of modules.
- 4, Local rings and Modules, Jordan Canonical forms.

#### **Unit-5: Ordinary and Partial Differential Equations**

1. **Second order linear equations:** Fundamental solution of the homogeneous equation, linear independence, reduction of order, linear equations with constant coefficients, method of undetermined coefficients, method of variation of parameters.
2. **System of first order equations:** solution of linear system, basic theory of system of first order linear equations, linear homogeneous system, fundamental matrices, non homogeneous linear system.
3. **Elementary Partial Differential Equations:** Definitions, equations solvable by direct integration, nonlinear equations of first order, Charpit's method,
4. **Single first order PDEs:** Semi linear and quasilinear equations, characteristics Cauchy problem, general solutions. Classification of second order PDEs.

#### **Unit-6: Mathematical Methods**

1. **Laplace Transforms:** Definition, notation, sufficient condition for existence, properties of L.T., Initial value and final value theorems. Methods of finding L.T., Definition of Inverse Laplace transform uniqueness and properties, Convolution theorem. Methods of finding inverse L.T., Applications of L.T. to ODE and PDE.
2. **Fourier Transforms:** The Fourier Integral, complex form of Fourier Integrals and Fourier Integral theorem; Fourier transforms; Fourier Cosine and Sine Transforms, Finite Fourier transforms, Convolution theorem, Parseval's Identity.
3. **Linear Boundary value problems:** Introduction, wave equation, heat equation and Laplace's equation in Cartesian, Cylindrical and Spherical coordinates. Principle of superposition, series solutions, Fourier series, Solution of Linear boundary value problem by separation of variables, types of initial value problems and general solution of partial differential equations. Sturm-Liouville problems.

4. **Difference Equations:** Definition, order, linear difference equation, formation, existence and uniqueness theorem, solution of the equation, general solution of the homogenous difference equations and non-homogenous linear difference equations.

### Reference Books:

#### Unit-1:

1. Savita Arora and S. C. Malik: *Advanced Analysis*, New Age **International** Ltd.
2. Robert G. Bartle and Donald R. Sherbert: *Introduction to Real Analysis*, John Wiley and Sons.
3. R. R. Goldberg: *Methods of Real Analysis*, New Age International Ltd,

#### Unit-2:

1. J. B. Conway: *Functions of one complex variable*, Springer-Verlag.
2. S. Ponnusamy: *Foundation of Complex Analysis*, Narosa Publishing House.
3. Murray R. Spiegel: *Complex Variables*, Schaum's Outline Series,

#### Unit-3:

1. David S. Dummit and Richard M. Foote: *Abstract Algebra*, John Wiley and Sons.
2. J. B. Fraleigh: *A first course in Abstract Algebra*, Narosa publishing House.
3. I. N. Herstein: *Topics in Algebra*, Wiley - Eastern.
4. V. K. Khanna and S. K. Bhambri: *Abstract Algebra*, Vikas Publication.
5. I. S. Luther and L B. S. Passi: *Algebra-Groups- Volume /*, Narosa Publishing House.

#### Unit-4:

1. N.S.Gopalkrishna : *University Algebra*, Wiley-Eastern 1988.
2. N.S.Gopalkrishna : *Commutative Algebra*, Oxonian Press 1988.
3. Serge Lang : *Linear Algebra*, Addition-Wesley Publishing company.

4. Seymour Lipschutz : *Theory and Problems of linear Algebra*, Schauta's outline series.
5. I. S. Luthar and I. B. S. Passi: *Algebra, Volume 3; Modules*, Narosa Publishing House,

#### **Unit-5:**

1. William E. Boyce and Richard C. DiPrima: *Elementary Differential Equations and Boundary Value Problems*, Second ...Edition, John Wiley and. Sons, Inc., New York.
2. Earl A. Coddington: *An Introduction to Ordinary Differential Equations*, Prentice-Hall India, 2001.
3. H. K. Dass: *Advanced Engineering Mathematics*, S. Chand and Company Ltd., New Delhi 1999.
4. Fritz John; *Partial Differential Equations*, Forth edition, Springer International edition, New York 2009.
5. Robert C. McOwen: *Partial Differential Equations Methods and Applications*, Pearson Education Inc., Indian branch Delhi, 2004.
6. Phoolan Prasad and Renuka Ravindran: *Partial Differential Equations*, New age International Publishers, New Delhi 2005.
7. George F. Simmons: *Differential Equations with Applications and Historical Notes*, Tata McGraw-Hill Publications, New Delhi 1985.

#### **Unit-6;**

1. J. P. Chauhan: *Differential and Difference equations*, Garg Publishing House.
2. R. V. Churchill and J. W. Brown; *Fourier Series and Boundary value problems*, Mc Graw-Hill international.
3. Goyal and Gupta; *Laplace transforms and Fourier transforms*, Pragati Prakashan,
4. Murray J. Spiegel; *Laplace Transforms*, Schauta's outline Series.

## Paper-3 Special Paper (Select any one of the following papers).

(Course work of PhD)

### 1. METHODS OF WEIGHTED RESIDUES AND COLLOCATION METHOD

1. Collocation method.
2. Sub-domain method.
3. Least Squares method.
4. Galerkin method and Method of moments

Reference Book:

1. Grandin, H., Fundamentals of the Finite Element Method, Waveland Press, 1991
2. M.K. Jain, S.R.K. Iyenger, R.K. Jain: "Numerical Methods for scientific and engineering computations", VI – edition, New Age International Publishers

### 2. SYMMETRIES OF DIFFERENTIAL EQUATIONS.

1. Dimensional Analysis, Modelling, and Invariance
2. Dimensional Analysis—Buckingham Pi-Theorem
3. Assumptions Behind Dimensional Analysis
4. Application of Dimensional Analysis to Partial Differential Equations

Reference Book

1. Symmetries and Differential Equations , George W Bluman Sukeyuki Kumei, Springer-Verlag, 1989
2. P. E. Wellstead : Introduction to Physical System Modeling, Academic Press,1979.

### 3. NUMERICAL SOLUTIONS OF BOUNDARY VALUE PROBLEMS AND FEM

1. The Shooting Method, Solution through a set of equations, Derivative Boundary Conditions,
2. Rayleigh-Ritz, Galerkin Methods,
3. The Finite-Element Method.
4. Examples.

Reference Books

1. Gourdin and Boumhart: "Applied Numerical Analysis", P.H.I., 1996
2. R. J. Leveque: Numerical methods for conservation Laws, Birkhauser Verlag, Basel, 1992

### 4. ELEMENTARY NUMBER THEORY

**Unit – I:**

Basic concepts of divisibility, Euclidean algorithm, Prime numbers, Modular Arithmetic, Congruences, Fermat's theorem and Euler's generalization, The Greatest Integer Function. (8 hours)

**Unit – II:**

Continued Fractions: Finite Continued fractions (CF), Convergent of CF, Solving linear Diophantine equations using CF, Infinite CF, Representation of irrational number as an infinite CF and vice versa. (8 hours)

**Unit – III:**

Diophantine equations: Solutions of Diophantine equations  $ax + by = c$ ,  $x^2 + y^2 = z^2$ ,  $x^4 + y^4 = z^2$ ,  $x^4 - y^4 = z^2$ , Solution of Pell's equation  $x^2 - dy^2 = 1$  by direct method as well as by using Continued fractions. (8 hours)

**Unit – IV:**

Fibonacci sequence: Introduction, some simple properties of Fibonacci numbers, Number-theoretic properties of Fibonacci numbers, Certain identities involving Fibonacci numbers, Golden proportion, Fibonacci numbers and Continued fractions. (8 hours)

**References :**

1. David M. Burton : *Elementary Number Theory*, Tata McGraw–Hill Pub. Co. Ltd., N. Delhi, 6<sup>th</sup> edn, 2006.
2. S. G. Telang : *Number Theory*, Tata McGraw Hill Pub. Co. Ltd., New Delhi, 1996.
3. Titu Andreescu, Dorin Andrica, Ion Cucurezeanu : *An Introduction to Diophantine Equations*, Birkhäuser, 2010.
4. Koshy Thomas : *Fibonacci and Lucas Numbers with applications*, John Wiley and Sons, Inc., N. York, 2001.