

DDD-0215

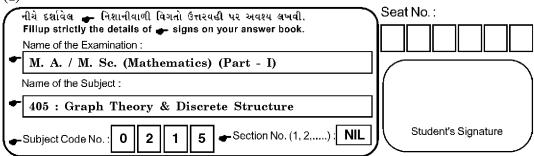
M. A. / M. Sc. (Mathematics) (Part - I) Examination March / April - 2016

405 : Graph Theory & Discrete Structure

| Time: Hours] | [Total Marks : 100 |
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Instructions:

(1)



- (2) Attempt all questions.
- (3) Figures to the right indicate marks.
- (4) Follow the usual notations and conventions.
- 1 (a) Prove that a given connected graph G is an Euler graph iff all vertices of G are of even degree.
 - (b) Prove that an Euler graph G is arbitrarily traceable from vertex v in G iff every circuit in G contains v.

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(c) Show that in a simple graph with n vertices the maximum degree of any vertex is n-1 and the maximum number of edges is $\frac{n(n-1)}{2}$.

OR

- 1 (a) Prove that any connected graph with n vertices and n-1 edges is a tree.
 - (b) Prove that in a complete graph with n(≥3) vertices, 7
 there are n = n-1/2 edge disjoint Hamiltonian circuits where n is an odd number.
 - (c) Discuss the Utilities Problem.

- 2 (a) Prove that Kuratowski's first graph is non-planar.
 - (b) Prove that a graph can be embedded in a surface of the sphere iff it can be embedded in a plane.
 - (c) Prove that a tree with more than two vertices will always has at least two pendant vertices.

OR

- 2 (a) What is the minimum possible height of an n-vertex 7 binary tree?
 - (b) Discuss the observation about the adjacency matrix. 7
 - (c) Prove that in a connected graph G, the complement of a cut-set in G does not contain a spanning tree and the complement of a spanning tree does not contain a cut-set.
- 3 (a) Prove that the ring sum of any two cut-sets in a 7 graph is either a third cut-set or an edge disjoint union of cut-sets.
 - (b) Let f from $\langle X, 0 \rangle$ onto $\langle Y, \oplus \rangle$ be a homomorphism 7 then prove that E_f is a congruence relation on $\langle X, 0 \rangle$ given by $x_1 E_f x_2 \Leftrightarrow f(x_1) = f(x_2)$ for any $x_1, x_2 \in X$.
 - (c) Let X be a non-empty set then prove that $\langle X^X, 0 \rangle$ **6** is a monoid, where $(f \circ g)(x) = f(g(x))$ for $f, g \in X^X$ and $x \in X$.

OR

3 (a) Define cancatenation and prove that for any alphabet 7 $V, \langle V^*, 0 \rangle$ is a monoid, where 'o' represents cancatenation.

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| (b) | Let B and A be the circuit and incidence matrix respectively, whose columns are arranged using the same order of edges. Then every row of B is orthogonal to every row of A. | 7 | |
|-----|--|---|--|
| (c) | Prove that any subgraph g of a connected graph G is contained in some spanning tree of G iff g contains no circuits. | 6 | |
| (a) | For the set of natural numbers N , prove that $\langle N, + \rangle$ | 7 | |
| | is a semigroup. Is the set odd non-negative integers | | |
| | form a subsemi group for $\langle N, + \rangle$? Justify your answer. | | |
| (b) | Prove that $\langle Z_m, +_m \rangle$ and $\langle Z_m^*, \oplus_m \rangle$ are isomorphic. | 7 | |
| (c) | Prove that for a semigroup homomorphism commutativity is preserved. | 6 | |
| OR | | | |
| (a) | Use the tabular representation and circuit diagram representation to represent $f = \sum (0,1,2,3,13,15)$. | 7 | |
| (b) | Define partial ordering relation. Give an example with justification. | 7 | |
| (c) | Prove that the rank of a well-formed polish notation is 1. | 6 | |
| (a) | Obtain the minimal expression for the function $f = \Sigma(5,7,10,13,15)$ | 7 | |
| a \ | Using Karnaugh map and Quine Mc-clusky algorithm. | _ | |
| (b) | Define minterm. Obtain the sum of product and | 7 | |

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(c)

a, b and c.

OR

product of sum canonical form for a * b in the variables

Prove that the modular inequality holds in a lattice.

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- 5 (a) Prove that the operations of meet and join on a lattice are commutative, associative, idempotent and satisfies the law of absorption.
 - (b) Prove that the direct product of two distributive lattices 7a distributive lattice.
 - (c) Show that, in a lattice 0 is the only complement of 1. 6

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