2303080304020002
EXAMINATION APRIL 2024
MASTER OF SCIENCE (MATHEMATICS)
(FOURTH SEMESTER)
ADVANCED LINEAR ALGEBRA

[**Time: Three Hours]** **[Max. Marks: 70]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **MASTER OF SCIENCE (MATHEMATICS) (FOURTH SEMESTER)**
   b. Name of the Subject : **ADVANCED LINEAR ALGEBRA**
   c. Subject Code No : **2303080304020002**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. There are five questions in this question paper

Seat No:

Student's Signature

Q.1 Attempt any Two. **14**

A. Define Homomorphism. If V and Ware of dimension m and n, respectively then prove that Hom (V,W) is of dimension mn over F.

B. Define Annihilator. If V is finite dimensional and W is subspace of V, then prove that $\widehat{W}$ is isomorphic to $\frac{\hat{V}}{A(W)}$ and dim A(W) = dim *V* - dim *W*.

C. Define dual space. If V is finite dimensional, then prove that ψ is an isomorphism of V onto $\hat{V}$.

Q.2 Attempt any Two. **14**

A. Define Algebra. Find rank and nullity of system of equations.

$$x_1 + 2x_2 - 3x_3 + 4x_4 = 0$$
$$x_1 + 3x_2 - x_3 = 0$$
$$6x_1 + x_3 + 2x_4 = 0$$

B. Define Range. If V is finite dimensional over F, then prove that Tϵ A(V) is regular if and only if T maps V onto V.

C. If V is finite dimensional over F then for S,T ϵ A(V),
(1) r(ST) ≤ r(T);

(2) r(TS) ≤ r(T);
(3) r(ST) = r(TS) = r(T).

Q.3 Attempt any Two. **14**

A. Let V be finite dimensional vector space over F. If T∈ A(V) has all its characteristic roots in F then there is a basis for V over F in which the matrix of T is triangular.

B. Define characteristic root. If λ_1, ..., $\lambda_k$in F are distinct characteristic roots of T ∈ A(V) and $v_1, v_2, \ldots . v_k$ are characteristic vectors of T belonging to λ_1,..., λ_k respectively then $v_1, v_2, \ldots . v_k$ are linearly independent over F.

C. If V is n-dimensional over F and if T ∈ A(V) has all its characteristic roots in F then prove that T satisfies a polynomial of degree n over F.

Q.4 Attempt any Two. **14**

A. Define Nilpotent transformation. If T ∈ A(V) is nilpotent then,$\alpha_0 + \alpha_1 T +$ ... + $\alpha_m T^m$; $\alpha_i's \in$ F is invertible then show that $\alpha_0 \neq 0$.

B. Define cyclic module. Prove that in a finite dimensional vector space, invariants of nilpotent transformation are unique.

C. Find the rational canonical form of the linear transformation T ∈ R^2 with minimal polynomial p(x) = (x − 1)$(x^2 + 1)^2$

Q.5 Attempt any Two. **14**

A. If W ⊂ V is invariant under T, then prove that T induces a linear transformation $\bar{T}$on $\frac{v}{w}$ defined by (v +W) $\bar{T}$ =vT+W. Also, if T satisfies the polynomial q(x) ∈ F[x] then prove that $\bar{T}$ also satisfies the same polynomial. If p_1(x) is a minimal polynomial for $\bar{T}$ over F and if p(x) is that for T then prove that p_1(x)/p(x).

B. Define Jordan form. Find the Jordan form of A=$\begin{bmatrix} 0 & -4 \\ 1 & 4 \end{bmatrix}$, Obtain Jordan block which is similar to $P^{-1}AP$.

C. Prove that there exists a subspace W of V_1, invariant under T, such that V = $V_1 \oplus$W.

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