2003080104040003

EXAMINATION APRIL 2024

MASTER OF SCIENCE (PHYSICS) (FOURTH SEMESTER)

GROUP THEORY AND QUANTUM FIELD THEORY

[**Time: Three Hours]** **[Max. Marks:70]**

Instructions:

1. Fill up strictly the following details on your answer book

a. Name of the Examination : **MASTER OF SCIENCE (PHYSICS) (FOURTH SEMESTER)**

b. Name of the Subject : **GROUP THEORY AND QUANTUM FIELD THEORY**

c. Subject Code No : **2003080104040003**

2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Symbols used have their usual meaning
6. Non-programmable scientific calculator may be used.

Seat No:

Student's Signature

Q.1 Attempt any two questions.

(i) a) Define direct product of groups. Find the direct product of $(E, m_x)\otimes(E, m_y)$ where $E, m_x m_y, \in C_{4v}$. **3**

b) Define a matrix representation of a group and list the properties of such representation. **3**

(ii) a) Show that the set of all non-singular matrices of order n form a group under matrix multiplication. **3**

b) Discuss the isomorphism and homomorphism. **3**

(iii) State the orthogonality theorem for irreducible representations and prove it using Schur's lemma. **6**

Q.2 Attempt any two questions.

(i) a) What is a Casimir operator? Find the Casimir operator(s) of group SO(3). **3**

b) What is meant by parameter and rank of a continuous group? **3**

(ii) a) Express a general element of a Lie group with r continuous parameters in terms of the corresponding generators. **3**

b) How many generators are needed for group SU(3)? What is the rank of this group and how many Casimir operators are there for this group? **3**

(iii) Discuss the axial rotation group SO(2). **6**

Q.3 Attempt any two questions.

(i) a) Show that the Lagrange density given by $L = \frac{1}{2}\emptyset^2 - \frac{1}{2}(\nabla\emptyset)^2 - \frac{1}{2}m^2\emptyset^2$ satisfies the Klein Gordon equation. **3**

b) Discuss the Klein-Gordon equation for a free particle and mention the difficulties with this equation. **3**

(ii) a) What is meant by x^μ; ∂_μ; box operators? **3**

b) Derive the Dirac equation for the ψ field from the Lagrange density given by $L = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$. Discuss quantization conditions. **3**

(iii) Discuss the interaction of neutral pions with nucleons and hence find an expression for the Yukawa potential. **6**

Q.4 Attempt any two questions.

(i) Describe the initial and final state vectors for the following processes and draw the corresponding Feynman diagrams: **6**

a. Electron-electron scattering;
b. Electron muon scattering;
c. Electron-proton scattering.

(ii) a) Identify the processes corresponding to the following terms and draw the corresponding Feynman diagrams: **3**

$a^{\dagger}(k)c^{\dagger}(p')c(p), a^{\dagger}(k)c(p)d(p)$.

b) What is meant by normal product and contraction? **3**

(iii) a) Establish the anti-commutation relations for the Dirac field. **3**

b) Define normal product operator N and find $N\left(\varphi^{(+)}(x)\varphi^{(-)}(y)\right)$. **3**

Q.5 Attempt any two questions.

i. Discuss non-Abelian transformations of gauge fields. **6**

ii. What is difference between the global and local gauge transformation of a complex scalar field? Is the free Lagrangian density invariant under global gauge transformation? If not then explain how the gauge invariance is restored in case of local gauge transformation? **6**

iii. Elaborate on the U(1) gauge symmetry and its role in Abelian transformations? Additionally, explain how gauge invariance necessitates the vector meson to be massless. **6**

Q.6 Attempt any two questions.

i. Write a brief note on Quantum Chromodynamics. **5**

ii. Discuss electroweak theory of Leptons in the Standard model. **5**

iii. State the difference between QCD and QED. **5**

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