2303080303040006
EXAMINATION JUNE- 2025
MASTER OF SCIENCE (MATHEMATICS)
(THIRD SEMESTER)
ADVANCED SPECIAL FUNCTIONS - I

[Time: As Per Schedule] **[Max. Marks: 70]**

Instructions:
1. **Fill up strictly the following details on your answer book**
   a. Name of the Examination : **MASTER OF SCIENCE (MATHEMATICS) (THIRD SEMESTER)**
   b. Name of the Subject : **ADVANCED SPECIAL FUNCTIONS - I**
   c. Subject Code No : **2303080303040006**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.

Seat No:

Student's Signature

Q.1 Answer any <u>TWO</u> of the following **14**

(I) Obtain the contiguous function relation

(i) $(a_1 - b_k + 1)F = a_1 F(a_1+) - (b_k - 1)F(b_k-)$

(ii) $a_1 F = a_1 F(a_1\ +) - x\ \sum_{j=1}^{q} U_j F(b_j\ +), p < q$

(II) If n is a non-negative integer and if a, b, c are independent of n, then prove that, ${}_3F_2\left[\begin{matrix} -n, & a, & b \\ c, 1-c+a+b-n & & \end{matrix}\ ;\ 1\right] = \frac{(c-a)_n (c-b)_n}{(c)_n (c-a-b)_n}$

(III) Show that $w = {}_pF_q$ is a solution of the Generalized Hypergeometric differential equation

$$\left[\theta \prod_{j=1}^{q}(\theta + b_j - 1) - z\prod_{i=1}^{p}(\theta + a_i)\right] w = 0$$

Q.2 Answer any <u>TWO</u> of the following: **14**

(I) If n is a non negative integer and if b and c are independent of n, show that

$$ {}_3F_2\left[\begin{matrix} -n, & b, & c; \\ 1-b-n, & 1-c-n; & \end{matrix} \middle| x\right] $$

$$ = (1-x)^n\, {}_3F_2\left[\begin{matrix} \frac{-n}{2}, \frac{-n}{2}+\frac{1}{2}, 1-b-c-n; \\ 1-b-n, \quad 1-c-n; \end{matrix} \middle| \frac{-4x}{(1-x)^2}\right] $$

(II) If Re(z) and if number a_m or b_j is not zero or a negative integer then

$$ \frac{1}{2\pi i}\int_{B_n} \frac{(-z)^s \Gamma(-s) \prod_{m=1}^{p} \Gamma(a_m+s)}{\prod_{j=1}^{q} \Gamma(b_j+s)} ds = \frac{\prod_{m=1}^{q} \Gamma(a_m)}{\prod_{j=1}^{q} \Gamma(b_j)}\, {}_qF_q\left[\begin{matrix} a_1, a_2, \ldots, a_q; \\ b_1, b_2, \ldots, b_q \quad ; \end{matrix} \middle| z\right] $$

(III) Show that If $p = q+1$ and no a_m is zero or a negative integer show that

$$ \frac{1}{2\pi i}\int_{B} \frac{\Gamma(-s)\ (-z)^s \prod_{m=1}^{p} \Gamma(a_m+s)}{\prod_{j=1}^{q} \Gamma(b_j+s)}\ ds\ ; $$

where B is Barnes contour ; is an analytic function of z in the finite plane $|are(-z)| < \pi$

Q.3 Answer any <u>TWO</u> of the following: **14**

(I) State and prove the Kummer's second formula.

(II) For the function $e^t\psi(xt) = \sum_{n=0}^{\infty} \sigma_n(x)t^n$, $\psi(u) = \sum_{n=0}^{\infty} \gamma_n u^n$ show that $(1-t)^{-c}F\left(\frac{xt}{1-t}\right) = \sum_{n=0}^{\infty}(c)_n\, \sigma_n(x)t^n$, where C is arbitrary In which $F(u) = \sum_{n=0}^{\infty}(c)_n\, \gamma_n u^n$.

(III) With usual notation prove that

$$ \mathrm{zJn'(z)} = \mathrm{zJ_{n-1}(z)} - \mathrm{nJ_n(z)} $$
$$ \mathrm{zJn'(z)} = -\mathrm{zJ_{n+1}(z)} + \mathrm{nJ_n(z)} $$

Q.4 Answer any <u>TWO</u> of the following: **14**

(I) Let $\psi(u) = \sum_{n=0}^{\infty} \gamma_n u^n$ and $(1-t)^{-c} F\left(\frac{-4xt}{(1-t)^2}\right) = \sum_{n=0}^{\infty} f_n\,(x) t^n$, then show that $f_n(x) = \frac{(c)_n}{n!} \sum_{k=0}^{n} \frac{(-n)_k (C+n)_k \gamma_k x^k}{\left(\frac{c}{2}\right)_k \left(\frac{c}{2}+\frac{1}{2}\right)_k}$

(II) Let the polynomials $f_n(x)$ are defined by $(1-t)^{-c} \psi\left(\frac{-4xt}{(1-t)^2}\right) = \sum_{n=0}^{\infty} f_n\,(x) t^n$ where $\psi(u) = \sum_{n=0}^{\infty} \gamma_n u^n$, $\gamma_0 \neq 0$ then show that $x f'_n(x) - n f_n(x) = -c \sum_{k=0}^{n-1} f_k(x) - 2x \sum_{k=0}^{n-1} f'_k(x)$; $n \geq 1$

(III) If $A(t) \exp\left(\frac{-xt}{1-t}\right) = \sum_{n=0}^{\infty} y_n(x) t^n$ then prove that $y_n^1(x) = y_{n-1}^1(x) - y_{n-1}(x)$ and $y_n'(x) = -\sum_{k=0}^{n-1} y_k(x)$, $n \geq 1$

Q.5 Answer any <u>TWO</u> of the following: **14**

(I) Obtain the values of $J_{\frac{1}{2}}(z)$ and $J_{-\frac{1}{2}}(z)$

(II) For integral n show that $J_n(z) = \frac{1}{\pi} \int_0^x Cos(n\theta - z \sin\theta) d\theta$

(III) Show that the Neumann polynomial's defined by $0_0(s) = s^{-1}$, $0_1(s) = s^{-2}$, $0_n(s) = 0_{n-2}(s) - 2\,0'_{n-1}(s)$, $n \geq 2$. are given by $O_0(s) = s^{-1}$, and $O_n(s) = \frac{n}{4} \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{(n-1-k)!}{k!} \left(\frac{2}{s}\right)^{n+1-2k}$; $n \geq 1$

*****